1. A two-slit Fraunhofer interference-diffraction pattern is observed using light that has a wavelength equal to 500 nm. The slits have a separation of 0.100 mm and an unknown width. (a) Find the width if the fifth interference maximum is at the same angle as the first diffraction minimum. (b) For this case, how many bright interference fringes will be seen in the central diffraction maximum?

**Solution:**

Write down what is known, and what is being asked for: $\lambda = 500 \times 10^{-9} m$, $d = 0.100 \times 10^{-3} m$, $a =?$

Now we call upon the physics: The interference maxima are located at $d \sin\theta = m\lambda$ where $m = 0, 1, 2, 3, \ldots$. The diffraction dark spots are located at $a \sin\theta = n\lambda$.

For part a, we set up the equation:

$$d \sin\theta = 5\lambda \quad \theta = \sin^{-1}\left(\frac{5(500 \times 10^{-9} m)}{0.100 \times 10^{-3} m}\right) = 1.43^\circ$$

The diffraction equation is

$$a \sin\theta = (1)\lambda \ \rightarrow \ a = \frac{\lambda}{\sin\theta} = \frac{500 \times 10^{-9} m}{\sin(1.43^\circ)} = 2.00 \times 10^{-5} m$$

Since the first minima occurs on the fifth bright fringe, then there will be 4 fringes on either side of the central bright fring for a total of nine bright fringes in the central group (the 1 central fringe, + 4 on the left side and + 4 on the right side).

2. Suppose that the central diffraction maximum for two slits has 17 interference fringes for some wavelength of light. How many interference fringes would you expect in the diffraction maximum adjacent to one side of the central diffraction maximum?

**Solution:** This is a ratios problem. If there are 17 central fringes, then that means there are 8 on either sideo of the one central fringe. So the first diffraction minimum is $n = 1$ and that coincides with $m = 9$. The ratio of the interference and diffraction equations:

$$\frac{d \sin\theta = m\lambda}{a \sin\theta = n\lambda}$$

gives,

$$\frac{d}{a} = \frac{9}{1} \quad d = 9a$$

Then for the second diffraction dark zone:

$$a \sin\theta_2 = 2\lambda = \frac{d}{9} \sin\theta_2 = 2\lambda$$

This corresponds to the 18th fringe of the interference pattern:

$$d \sin\theta_2 = 18\lambda$$

Remember that the 9th fringe is darkened, and now we know that the 18th fringe is darkened, so there is 8 non-darkened fringes between those two darkened fringes (10, 11, 12, 13, 14, 15, 16, 17). The answer is thus 8.

3. The colors of many butterfly wings and beetle carapaces are due to the effects of diffraction. The Morpho butterfly, for example, has structural elements on its wings that effectively act as a diffraction grating that has an 880-nm spacing. At what angle will the first diffraction maximum occur for normally incident light diffracted by the butterfly's wings? Assume the light is blue and has a wavelength of 440nm.

**Solution:** Using

$$d \sin\theta = m\lambda$$

with $m = 1$, $d = 880 \times 10^{-9}m$, $\lambda = 440 \times 10^{-9}m$, we solve for $\theta$ via inverse sine to find:

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = 30.00^\circ$$

4. The red light from a helium-neon laser has a wavelength of 632.8 nm in air. Given $n_{air} = 1.00$, $n_{H_2O} = 1.33$, and for this particular glass, $n_g = 1.50$.

(a) Find the speed of this light in air, water, and glass.

**Solution:** Whereas the speed of light in a vacuum is $c = \left(2.99 \times 10^8\right) \frac{\mathrm{m}}{\mathrm{s}}$ and in a medium, $v = c/n$, we have:

$$\begin{aligned} v_{air} &= \frac{c}{1.00} = \left(2.99 \times 10^8\right)\frac{\mathrm{m}}{\mathrm{s}} \\ v_{H_2O} &= \frac{c}{1.33} = 2.25 \times 10^8 \frac{m}{s} \\ v_g &= \frac{c}{1.50} = 1.99 \times 10^8 \frac{m}{s} \end{aligned}$$

(b) Find the wavelength of this light in air, water, and glass.

**Solution:** We recall that the frequency doesn't change in a medium, the wavelength does, and so we have:

$$\begin{aligned} \lambda_{air} &= \frac{\lambda_0}{1.0} = 632.8nm \\ \lambda_{H_2O} &= \frac{\lambda_0}{1.33} = 4.76 \times 10^2 nm \\ \lambda_g &= \frac{\lambda_0}{1.33} = 4.22 \times 10^2 nm \end{aligned}$$

(c) Find the frequency of this light in air, water, and glass.

**Solution:** Since frequency doesn't change with medium, then all cases will have the same frequency:

$$f = \frac{c}{\lambda_0} = 4.73 \times 10^5 Hz$$

5. A slab of glass that has an index of refraction of 1.50 is submerged in water that has an index of refraction of 1.33. Light in the water is incident on the glass. Find the angle of refraction if the angle of incidence is 60°, 45°, and 30°. **Given:** $n_{air} = 1.00$**.**

**Solution:** For each case, we need to use Snell's Law:

$$n_{H_20}\sin(\theta_i) = n_g\sin(\theta_r)$$

and solve for

$$\theta_r = \sin^{-1}(\frac{n_{H_20}}{n_g}\sin(\theta_i)$$

so for example:

$$\theta_r = \sin^{-1}(\frac{1.33}{1.55}\sin(60)) = 48.0^\circ$$

$$\theta_r = \sin^{-1}(\frac{1.33}{1.55}\sin(45)) = 37.4^\circ$$

$$\theta_r = \sin^{-1}(\frac{1.33}{1.55}\sin(30)) = 25.4^\circ$$

6. A point source of light is located 5.0 m below the surface of a large pool of water. Find the area of the largest circle on the pool's surface through which light coming directly from the source can emerge. *Hint: Search for the angle of total internal reflection.* **Given: The index of refraction of the air is 1.00 and of the water is 1.33**.

**Solution:** Beyond some angle, $\theta$, we will have total internal reflection. This happens because the index of refraction of the air above the water is greater than the index of refraction of the water itself (1.00 vs 1.33). The condition is that the refraction angle becomes 90 degrees relative to the normal to the surface–the point at which light bends parallel to the surface of water and thus doesn't escape:

$$n_{air}\sin(90) = n_{H_2O}\sin(\theta) \quad \rightarrow \quad \theta = \sin^{-1}\left(\frac{n_{air}}{n_{H_2O}}\right) = 48.8^\circ$$

Then using geometry, where the tangent is opposite over adjacent, we have:

$$r = (5.0m)\tan(48.8^\circ) = 5.7m$$

The are of the corresponding largest circle is:

$$A = \pi r^2 = 102.1m^2$$

7. When monochromatic ultraviolet light that has a wavelength equal to 300 nm is incident on a sample of potassium, the emitted electrons have a maximum kinetic energy of 2.03 eV. **Given:** $hc = 1240eV(nm)$.

   (a) What is the energy of an incident photon?

   **Solution:** The energy of a photon is $E = hf$ or

   $$E = \frac{hc}{\lambda} = \frac{1240eV(nm)}{\lambda} = \frac{1240}{300}eV = 4.13eV$$

   (b) What is the work function for potassium?

   **Solution:** Recall that

   $$KE_{max} = E_{photon} - \phi$$

   where $\phi$ is the work function. Here, we have:

   $$\phi = E_{photon} - KE = 4.13eV - 2.03eV = 2.103eV$$

   (c) What would be the maximum kinetic energy of the electrons if the incident electromagnetic radiation had a wavelength of 430 nm?

   **Solution:** Redo the above two equations and replace 300 nm with 430 nm to find:

   $$KE = E_{photon} - \phi = 2.884eV - 2.103eV = 0.780eV$$

   (d) What is the maximum wavelength of incident electromagnetic radiation that will result in the photoelectric emission of electrons by a sample of potassium?

   **Solution:** We take the limit for when the kinetic energy goes to zero and we have:

   $$0 = E_{photon} - \phi \;\rightarrow\; \frac{hc}{\lambda} = \phi$$

   In this case,

   $$\frac{1240eV(nm)}{\lambda} = 2.103eV \;\rightarrow\; \lambda = 5.90 \times 10^2 nm$$