Name: ________________________________

You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

Short answers

1. Why is c the ultimate speed limit in the universe?

2. The twin paradox is resolved by noticing that
   A. The twins are actually the same way at the end of the trip.
   B. The twin on the rocket is not in a preferred frame.
   C. The twin on Earth is not in the preferred frame.
   D. One twin accelerates which radically shifts their spacetime experience relative to the other twin.

3. How do we know that the universe is expanding?

4. What was the major clue for Einstein that Newtonian gravity was flawed?

Short Problems

5. Two motorcycle pack leaders named David and Emily are racing at relativistic speeds along perpendicular paths. Emily is traveling south at $v_y = -0.90c$ and David is traveling East as $v_x = +0.75c$ as measured by a police officer standing still in the middle of an intersection.

   How fast does Emily recede as seen by David over his right shoulder?

   Hint: It is often easier if you select one of the two frames of a problem to be one that is at rest relative to the problem (the cop) and the other frame to be the person whose perspective you are being asked to describe (David). In this case, Emily would be the third object whose velocity we wish to transform from the cop's frame to David's frame.

6. An electron ($m = 9.11 \times 10^{-31} kg$) moves with a speed of $0.1c, 0.5c$, and $0.9c$. In each case, find the percent difference between the relativistic momentum calculation and the classical momentum calculation.

7. The ^{216}Po nucleus is unstable and exhibits radioactivity. It decays to ^{212}Pb by emitting an alpha particle, which is a helium nucleus, ^{4}He. The relevant masses are $m_i = 216.001915u$ for ^{216}Po, $m_f = m(^{212}Pb) + m(^{4}He) = 211.991898u + 4.002603u$. $1u = 1.66x10^{-27}kg$.

   (a) Find the mass change in this decay.

   (b) Find the resulting release of energy.

8. The net nuclear fusion reaction inside the Sun can be written as $4^1H \rightarrow^4 He + E$. The rest energy of each hydrogen atom is $938.78MeV$, and the rest energy of the helium-4 atom is $3728.4MeV$.

   (a) Calculate the percentage of the starting mass that is transformed to other forms of energy.

   (b) If that energy were converted to the kinetic energy of a 1kg cannon ball, how fast would the cannon ball be traveling in meters per second?

Longer problems

9. A global positioning system (GPS) satellite moves in a circular orbit with period 11 h 58 min.

   (a) Determine the radius of its orbit. Recall from Physics I that for a circular orbit, the centripetal force and the gravitational force must balance so that

   $$\frac{G \cdot M_E \cdot m}{r^2} = \frac{mv^2}{r}$$

   where $G = 6.67 \times 10^{-11} Nm^2/kg^2$ is the original gravitational constant and the mass of the Earth is $M_E = 5.98 \times 10^{24} kg$. We also recall that $v = 2\pi r/T$ and substitution yields:

   $$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

   (b) Determine its speed. Recall that $v = 2\pi r/T$.

   (c) The nonmilitary GPS signal is broadcast at a frequency of $1575.42 MHz$ in the reference frame of the satellite. When it is received on the Earth's surface by a GPS receiver, what is the fractional change in this frequency due to time dilation as described by special relativity? The fractional change is $\Delta f/f$, i.e. the change in frequency divided by the original frequency.

   Hint: Our derivation for relativistic Doppler effect in class was only for when the receiver and/or source were moving along a straight line. In this case, the satellite is moving perpendicular to a receiver on Earth (circular motion) so we can instead use the time dilation equation equation. If the period of oscillation is ΔT on the satellite, then the period would be measured $\Delta T/\gamma$ here on Earth. Using $f = 1/\Delta T$ and $f' = 1/(\Delta T/\gamma)$, find $\Delta f/f = \frac{f-f'}{f}$.

(d) Calculate this fractional change in frequency due to the change in position of the satellite from the Earth's surface to its orbital position.

Given:

$$U_g = -\frac{GM_E m_s}{r}$$

$$\Delta U_g = U_g(r_s) - U_g(r_E)$$

and the radius of the Earth is $r_E = 6.37 \times 10^6 m$. You **will not** need to know the mass of the satellite, leave it as a variable m_s and it will cancel out eventually.

(e) What is the overall fractional change in frequency due to both time dilation and gravitational blueshift (add them up)? Which effect was bigger?

Some possibly useful equations.

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$
$$\Delta E_0^2 = \Delta mc^2$$
$$p = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}}$$
$$\mu_0 = \left(4 \times \pi \times 10^{-7}\mathrm{Tm/A}\right)$$
$$c = \left(2.99 \times 10^8\right)\frac{\mathrm{m}}{\mathrm{s}}$$
$$m_e c^2 = 0.511\ \mathrm{MeV}$$
$$K = (\gamma - 1)\, mc^2$$
$$K = E - mc^2$$
$$E^2 = p^2c^2 + (m_p c^2)^2$$
$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - \frac{vx}{c^2})$$
$$L' = \frac{L_p}{\gamma}$$
$$t' = \gamma t_p$$
$$\frac{\Delta f}{f} = \frac{\Delta U_g}{mc^2}$$
$$f = \sqrt{\frac{c+u}{c-u}} f_0$$
$$v_x' = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$$
$$v_y' = \frac{v_y}{\gamma\left(1 - \frac{v_x u}{c^2}\right)}$$
$$v_z' = \frac{v_z}{\gamma\left(1 - \frac{v_x u}{c^2}\right)}$$
$$K_{classic} = \frac{1}{2}mv^2$$

End of exam