Name: ______________________________

You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

Short answers

1. Proper length is
   - A. Found in the frame at rest relative to the Universe.
   - **B. Found in the rest-frame of the object or distance between events/objects that is being measured.**
   - C. Found in the frame in which the time of an incident would be the longest.
   - D. Found in the frame in which the object or distance between events/objects being measured is in motion.
   - E. I like apples.

2. What are the two postulates of relativity?

Solution:

1. There is no preferred reference frame; the laws of physics are the same in all frames.
2. It is a law of physics that the speed of light is $c = (2.99 \times 10^8)\,\frac{\mathrm{m}}{\mathrm{s}}$.

3. If you are measuring an astrophysical phenomenon, say an exploding star that triggers another star to explode, and your friend was doing so in a rocket traveling at speed 0.9c relative to you, in what way could your readings of the distance between the stars and the time between explosions differ?

Solution: Depending on the angle of my friend's travel, the time she measures the event to happen would be longer and the distance between the stars would appear shorter.

Short Problems

4. When Uranium-235 is hit by a neutron, it becomes Uranium-236 for a very short period. U-236 has a very short half-life and decays into Barium and Krypton (Ba-141 and Kr-92)

while releasing 3 neutrons, which means that we could have up to 3 more U-235 atoms hit resulting in 9 more projectile neutrons ready to split other atoms, and so on. That's the concept of a "chain-reaction".

Interestingly, the sum of the rest mass of the original particles (neutron plus U-235) is **more** than the sum of the rest mass of the outputted particles (Ba-141, Kr-92, and 3 neutrons). That missing mass was converted into energy.

Suppose that the energy released from a fission event (splitting described above) is 198 MeV, what is the corresponding change in mass? The conversion from electron volts (eV) to Joules is 1 eV $= 1.602 \times 10^{-19}$ J . Recall that $1J = N \cdot m$.

Solution: From the equation:

$$E = mc^2 \ \rightarrow \ \Delta E = \Delta mc^2$$

we have

$$\Delta m = \frac{\Delta E}{c^2} = \frac{198 \text{ MeV}}{\left((2.99 \times 10^8)\,\frac{\text{m}}{\text{s}}\right)^2} \left(\frac{1 \times 10^6 \cancel{eV}}{\cancel{MeV}}\right)\left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}}\right)$$

and finally,

$$\Delta m = 3.55 \times 10^{-28} \text{ kg}$$

5. What will be the half-life of a Mamahuhu particle as measured in a laboratory if the particle is seen traveling at $v = 0.9c$ with respect to the laboratory?

   For this particle, the half-life is 2.4×10^{-4} s .

Solution:

The proper frame for the lifetime of the Mamahuhu particle is in the Mamahuhu's frame. This is because we would measure the "birth" and decay (into something else) of the particle as events that happen at the same spot in that frame.

In the Mamahuhu's frame, the lifetime is 2.4×10^{-4} s . In a lab frame, the Mamahuhu's frame is moving at a speed of 0.9c. The lifetime a scientist in the lab would measure would be time-dilated as follows:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.4 \times 10^{-4} \text{ s}}{\sqrt{1 - \frac{8.10 \times 10^{-1} c^2}{c^2}}} = 5.51 \times 10^{-4} \text{ s}$$

Longer problems

6. (a) Find the rest energy of a **electron** in units of electron volts. Given: $m_e = 9.11 \times 10^{-31}$ kg , $c = (2.99 \times 10^8)\,\frac{\text{m}}{\text{s}}$, $1.00eV = 1.602 \times 10^{-19}J$.

Solution:

$$E_R = m_p c^2 = \left(9.11 \times 10^{-31}\text{ kg }\right)\left(\left(2.99 \times 10^8\right)\frac{\text{m}}{\text{s}}\right)^2\left(\frac{1.00eV}{1.602 \times 10^{-19}J}\right) = 5.08 \times 10^5\text{eV}$$

(b) If the total energy of a electron in this particular case is 6.0 times its rest energy, what is the speed of the electron relative to the lab from which it is launched?

Solution:

$$E = 6.0m_p c^2 = \gamma m_p c^2 \;\rightarrow\; \gamma = 6.0$$

And solving for v:

$$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} = \frac{1}{36.00}$$

Rearranging and solving for v:

$$v = 0.986c$$

(c) An event taking place in the electron's rest frame is clocked at 35.0 s, how long would that event seem to take in the lab frame? *Hint: The shortest time you can measure is in the rest frame*

Solution:

$$\Delta t' = \gamma \Delta t_p = \frac{35.0\text{s}}{\sqrt{1 - (0.986)^2}} = 210.00\text{ s}$$

(d) Determine the kinetic energy of this electron in units of electron volts.

Solution:

$$K = E - m_p c^2 = (6.0 - 1)\,5.08 \times 10^5\text{ eV} = 2.54 \times 10^6\text{eV}$$

(e) What is the electron's momentum?

Solution: From the equation,

$$E^2 = p^2c^2 + (m_p c^2)^2 = \left(6.0m_p c^2\right)^2$$

we can algebraically solve for p:

$$p = \sqrt{\frac{(6.0^2 - 1)\left(m_e c^2\right)^2}{c^2}} = 3.01 \times 10^6 \mathrm{eV/c}$$

Some possibly useful equations.

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\Delta E_0^2 = \Delta m c^2$$

$$p = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\mu_0 = \left(4 \times \pi \times 10^{-7} \mathrm{Tm/A}\right)$$

$$c = \left(2.99 \times 10^{8}\right) \frac{\mathrm{m}}{\mathrm{s}}$$

$$m_e c^2 = 0.511\ \mathrm{MeV}$$

$$K = (\gamma - 1)\, mc^2$$

$$K = E - mc^2$$

$$E^2 = p^2 c^2 + (m_p c^2)^2$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$L' = \frac{L_p}{\gamma}$$

$$t' = \gamma t_p$$