1. In the Stanford linear collider, small bundles of electrons and positrons are fired at each other. In the laboratory's frame of reference, each bundle is approximately 1.0 cm long ad 10 μm in diameter. In the collision region, each particle has an energy of 50 GeV, and the electrons and positrons are moving in opposite directions. Given:

$$E = \gamma mc^2$$

and for electrons/positrons,

$$mc^2 = 0.511 \text{ MeV}$$

find γ and proceed (this is from Relativistic Energy which we will cover in more detail next week).

a) How long and how wide is each bundle in its own reference frame?

b) What must be the minimum proper length of the accelerator for a bundle to have both its ends simultaneously in the accelerator in its own reference frame? (The actual proper length of the accelerator is less than 1000m.)

Solution: There is no motion in the direction of the width of the beam, and so the diameter here will remain the same. The direction of motion will see a length contraction in the laboratory frame.

We are given the total energy here, but not the speed. Since we hadn't gotten to relativistic energy, I gave you γ. Here, I'll show you how we got it.

The equation for relativistic total energy of a particle moving with velocity v is:

$$E = \gamma mc^2$$

and as would be given to you on a test, the rest energy of an electron is 0.511 MeV.

so then,

$$\gamma = \frac{E}{mc^2} = \frac{50.0 \times 10^9 \text{eV}}{0.511 \text{ MeV}} = 97847.4$$

Now we can solve for how long the bundle is in its rest frame. Recall that the width doesn't change, so we have to only consider the length. Also, **recall that the frame in which an object will be found to be at its longest length in the direction of motion is in its own rest frame, this is called the proper length.** Recall that the proper length equation is:

$$L' = \frac{L_p}{\gamma}$$

where L' is measured in a frame in motion relative to the rest frame of the object. and so in this case, where we have $L' = 0.01$ m,

$$L_p = \gamma L' = 978.5 \text{ m}$$

For part **b**, the wording here is a bit confusing so let's break it down. It is asking us to find out what the contracted length of the accelerator is in the frame of the beam. The condition is that the length must be greater than or equal to the proper length of the beam we just found above. That is, we want the length of the accelerator in the rest frame of the beam of electrons to be:

$$L'_a \geq 978.5 \text{ m}$$

$$L' = \frac{L_p}{\gamma} \ \rightarrow \ L_p = \gamma L' = (97847.4)\, 978.5 \text{ m } = 9.6 \times 10^7 \text{m}$$

The key here is that simultaneous events in one frame are not simultaneous in frames that are moving relative to that frame. Thus we would say in the lab frame that the beam is safely in the accelerator, that means we measured both ends to be in the lab at the same time, whereas according to the electron frame, those measurements happen at some time Δt which means that the beam, from its perspective, there is a time difference between when the head of beam exits the accelerator and when the tail of the beam enters the accelerator.

2. Unobtanium (Un) is an unstable particle that decays into normalium (Nr) and standardium (St) particles.

   (a) An accelerator produces a beam of Un that travels to a detector located $100m$ away from the accelerator. The particles travel with a velocity of $v = 0.866c$. How long do the particles take (in the laboratory frame) to get to the detector?

   (b) By the time the particles get to the detector, half of the particles decayed. What is the half-life of Un? (Note: half life as measured in rest frame of particles).

   (c) A new detector is going to be used, which is located 1000 m away from the accelerator. How fast should the particles be moving if half of the particles are to make it to the new detector?

Solution: Since all of the measurements happen **in the same frame**, we can use the equation $v = \Delta x/\Delta t \;\rightarrow\; \Delta t = \Delta x/t$, in this case

$$\Delta t = \frac{100\text{ m}}{0.866\left((2.99\times 10^{8})\,\frac{\text{m}}{\text{s}}\right)} = 3.9\times 10^{-7}\text{ s}$$

Proper time is the shortest amount of time that could be measured in any frame, and is always measured in the frame in which the event being clocked is at rest relative to the watch.

The equation that relates proper time to time in another frame:

$$\Delta t = \gamma \Delta t_p$$

Thus the half-life, which is always measured in the rest-frame of the particles, would be:

$$\Delta t_p = \frac{\Delta t}{\gamma} = 3.9\times 10^{-7}\text{ s}\sqrt{1-\frac{0.866^2c^2}{c^2}} = 1.9\times 10^{-7}$$

Finally, in order to find out the speed of the particles so that they can travel 1000 m and still make it in their half-life, then we need to find:

$$\frac{1000\text{ m}}{v}\sqrt{1-\frac{v^2}{c^2}} = 1.9\times 10^{-7}\text{ s}$$

$$1-\frac{v^2}{c^2} = \left(\frac{1.9\times 10^{-7}}{1000\text{ m}}\right)^2 v^2$$

Solve for $v = 0.998$c.

3. Observers in reference frame S see an explosion located on the $x-$axis at $x_1 = 480m$. A second explosion occurs 5.0 μs later at $x_2 = 1200m$. In reference frame S', which is moving along the $x-$axis in the $+x$ direction at speed v, the two explosions occur at the same point in space. What is the separation in time between the two explosions as measured in S'?

Solution: The Lorentz transform gives:

$$x' = \gamma(x - vt)$$

So then,

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right)$$

The separation between the events happens at the same point in space so that,

$$0 = \gamma\left(\Delta x - v\Delta t\right) \;\rightarrow\; \Delta x = v\Delta t \;\rightarrow\; v = \frac{1200.0\text{m} - 480.0\text{m}}{5.0\ \mu\text{s}} = 1.4 \times 10^8 \frac{\text{m}}{\text{s}} = 0.48c$$

Recall that the time transform is:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

so that

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

In our case:

$$\Delta t' = \frac{5.0\ \mu\text{s} - \frac{(0.48)(720.00)}{c}}{\sqrt{1 - (0.48)^2}} = 4.38 \times 10^{-6}\ \text{s}$$

4. **BONUS:** This is a qualitative question that results in one bonus percentage point.

   What will a square that is 5 meters by 5 meters at rest, is located 100 meters away from you, and is traveling at the following speeds, look like at speeds such as $v = 0.6c, v = 0.8c, v = 0.99c$?

   Think about the following factors: Length contraction, the time delay it takes for light from different parts of the square to reach your eye.