Name: ______________________________

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

1. A proton (charge $+e$, mass m_p), a deuteron (charge $+e$, mass $2m_p$), and an alpha particle (charge $+2e$, mass $4m_p$) are accelerated from rest through a common potential difference ΔV. Each of the particles enters a uniform magnetic field $\mathbf{B}$, with its velocity in a direction perpendicular to $\mathbf{B}$. The proton moves in a circular path of radius r_p. In terms of r_p , determine (a) the radius r_d of the circular orbit for the deuteron and (b) the radius r_a for the alpha particle.

Solution: The force from a magnetic field on a moving charged particle is given by,

$$\mathbf{F_B} = q\mathbf{v} \times \mathbf{B}$$

so that when the magnetic field is perpendicular to the velocity, we simply have the magnitude of the force is,

$$F_B = qvB$$

and the direction can be found using the right hand rule. It turns out this direction is perpendicular to the direction of velocity, which is the signature of a centripital forces that keep an object moving in a circle by changing the direction but not necessarily the magnitude of the velocity. This is what is happening in this problem, where we require that force provided by the magnetic field is the centripetal force we are looking for, i.e.,

$$qvB = \frac{mv^2}{R}$$

where R is the radius of the circle the particle will end up traveling in should it have an unobstructed path in the magnetic field, and should the magnetic field be in a large enough region to contain the circle. Thus

$$R = \frac{mv}{qB}$$

For this problem, we write:

$$r_p = \frac{m_p v_p}{eB}$$

where the velocity v is obtained by acceleration due to a common potential difference ΔV:

$$\frac{1}{2}mv^2 = e\Delta V$$

$$v = \sqrt{2e\Delta V/m_p}$$

Plug this into r_p to find:

$$r_p = \frac{\sqrt{2m_p\Delta V}}{\sqrt{e}B}$$

The deuteron has the same charge but twice the mass:

$$r_d = \frac{\sqrt{4m_p\Delta V}}{\sqrt{e}B} = \sqrt{2}r_p$$

The alpha particle has twice the charge and four times the mass:

$$r_\alpha = \frac{\sqrt{8m_p\Delta V}}{\sqrt{2e}B} = \frac{2}{\sqrt{2}}r_p = \sqrt{2}r_p$$

Surprisingly, the duetoron and the alpha particle will have the same cyclotron radius! **So then, suppose we have a bunch of deutorons and alpha particles, how can we separate them if they have the same cyclotron radius?**

2. Singly charged uranium-238 ions are accelerated through a potential difference of 2.00 kV and enter a uniform magnetic field of magnitude 1.20 T directed perpendicular to their velocities. (a) Determine the radius of their circular path. (b) Repeat this calculation for uranium-235 ions. (c) How could this be useful information to you if, say, you were a UN inspector trying to determine the ratio of U-238 to U-235 in Iran's refined uranium supply?

Solution: The mass of U-235 is 235.04 u, and the mass 238.05 u, where u is one atmoic mass unit and $1u = 1.66 \times 10^{-27}$. Let's assume that these are positive ions (they are missing one electron) so that they each have a charge of $+e$. Then their cyclotron radius (also called gyroradius or Larmor radius) is (as in the previous problem):

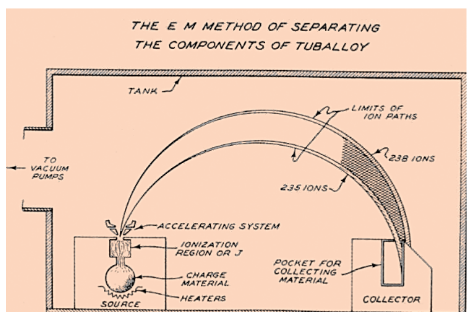
$$r = \frac{\sqrt{2m\Delta V}}{\sqrt{e}B}$$

The charge on a proton is 1.602×10^{-19} C, and so we have:

$$r_{235} = 0.08225m$$

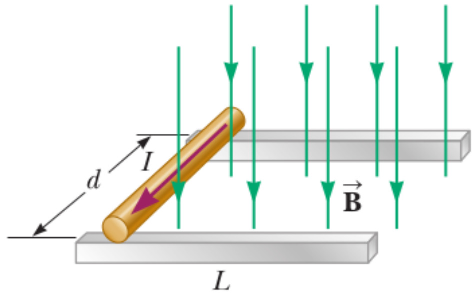
$$r_{238} = 0.08278m$$

This slight difference was proposed long-ago as one way to separate the two isotopes:



However, it isn't efficient enough for industrial level production; a high-end mass spectrometer could, on the other hand, use this effect to measure how much of a given uranium sample is the weapons-grade 235 verses the more common 238.

If this isn't a good way to isolate U-235 from U-238, what is a better way to do so?



3.

A rod of mass 0.720 kg and radius 6.00 cm rests on two parallel rails (see figure) that are d = 12.0 cm apart and L = 45.0 cm long. The rod carries a current of I = 48.0 A in the direction shown and rolls along the rails without slipping. A uniform magnetic field of magnitude 0.240 T is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod as it leaves the rails?

Solution: Recall that we can reformulate the magnetic force equation for currents as

$$F_B = I\mathbf{d} \times B$$

Where **d** is a length vector whose magnitude is the length of the rod $d = 12.0$ cm here, and whose direction is the direction of the current. The rod is free to role and it starts from rest, so evidently the magnetic force will provide a force that accelerates it to the right (right hand rule). The moment of inertia about a solid cylinder of radius r with the rotation axis about its center line is $I = mr^2/2$, and $a = r\alpha$.:

$$\tau_b = I\alpha \quad \rightarrow \quad F_b r = I\frac{a}{r} \quad \rightarrow \quad a = r\frac{F_b r}{I} = \frac{2IdB}{m} = 3.84\frac{\text{m}}{\text{s}^2}$$

The kinematic equations for motion with a uniform acceleration tell us that,

$$v_f = v_i + at \;\rightarrow\; t = \frac{v_f}{a} \text{ when } v_i = 0$$

$$x_f = x_i + v_i t + \frac{1}{2}at^2 \;\rightarrow\; \Delta x = \frac{1}{2}at^2 \text{ when } v_i = 0$$

$$\Delta x = \frac{1}{2}a\frac{v_f^2}{a^2} = \frac{1}{2}\frac{v_f^2}{a}$$

We want to solve for v_f:

$$v_f = \sqrt{2a\Delta x} = \sqrt{\frac{4IdBL}{m}} = 1.859\frac{\text{m}}{\text{s}}$$

But this is wrong! What have we done wrong here?

The actual solution can be solved much more quickly using the work-energy theorem:

$$\Delta W = \Delta K$$

Since $v_i = 0$, then,

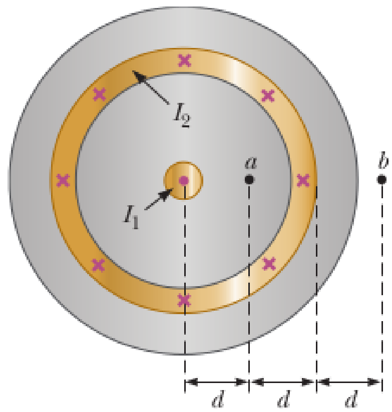
$$\Delta W = IdBL = K_{rot} + K_{trans} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}\frac{1}{2}mr^2\frac{v_f^2}{r^2} + \frac{1}{2}mv_f^2 = \frac{3Mv^2}{4}$$

Now we have:

$$v_f = \sqrt{\frac{4IdBL}{3m}} = 1.073\frac{\text{m}}{\text{s}}$$

which is the correct answer.

If you didn't treat it properly as a rod, you would have gotten 1.3 instead.

4. 

Behold a cross-section of Coaxial cable above. The center conductor is surrounded by a rubber layer, an outer conductor, and another rubber layer. In a particular application, the current in the inner conductor is $I_1 = 1.00$ A out of the page and the current in the outer conductor is $I_2 = 3.00$ A into the page. Assuming the distance d = 1.00 mm, determine the magnitude and direction of the magnetic field at (a) point a and (b) point b. *Hint: Use Ampere's Law.*

Solution: Form an Amperian loop around I_1 to find that at the point a where $r = 0.001$m,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$$

$$\int_0^{2\pi r} B \cdot dl = \mu_0 I_1 \quad \rightarrow \quad B = \frac{\mu_0 I_1}{2\pi r} = 2.000 \times 10^{-4}\text{T}$$

and at point b where $r = 0.003$ m, the enclosed current is $I_1 - I_2$, so,

$$\int_0^{2\pi r} B \cdot dl = \mu_0 I_1 \quad \rightarrow \quad B = \frac{\mu_0 (I_1 - I_2)}{2\pi r} = -1.333 \times 10^{-4}\text{T}$$

The direction of the field can be obtained by applying the right hand rule, with your thumb pointing in the direction of the current and the remaining fingers pointing in the direction of the magnetic field.

How can we obtain the same result using Biot-Savart Law?