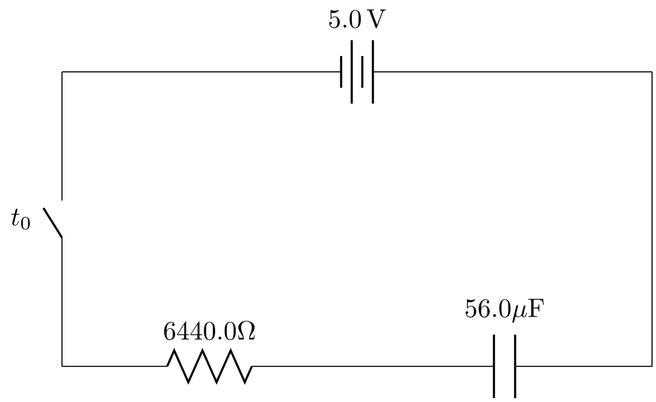
Name: ______________________________

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.



1. Find the charge, as a function of time, on the capacitor for the above circuit when the switch is closed. What is the value for each at 12 seconds? What about 120 seconds?

Solution: The charge on the capacitor is given by,

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) = 2.8 \times 10^{-4}\left(1 - e^{-t/\left(3.6 \times 10^{-1}\right)}\right)$$

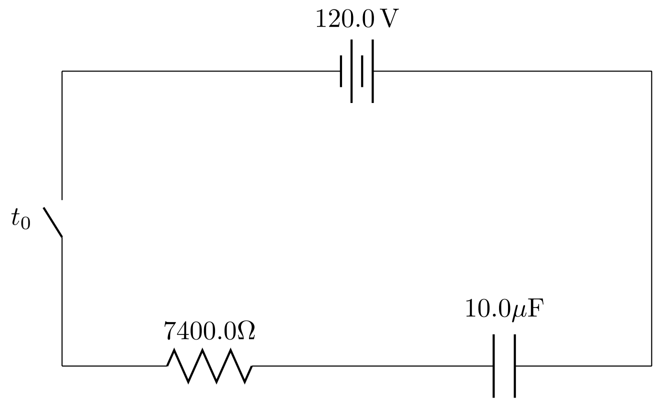
at 12 and 120 seconds:

$$Q(12) = C\mathcal{E}\left(1 - e^{-12/RC}\right) = 2.8 \times 10^{-4}\mathrm{C}$$

$$Q(120) = C\mathcal{E}\left(1 - e^{-120/RC}\right) = 2.8 \times 10^{-4}\mathrm{C}$$

2. Suppose that the circuit below is allowed to run for a very long period of time so that the capacitor is fully charged. Then the battery is removed and replaced by a wire so that the capacitor can discharge.

   Find the charge, as a function of time, and the current as a function of time. What is the value for each at 5 seconds? What about 50 seconds?



Solution: The charge on the capacitor (discharging) is given by,

$$Q(t) = C\mathcal{E}\left(e^{-t/RC}\right) = 1.2 \times 10^{-3}\left(e^{-t/\left(7.4\times 10^{-2}\right)}\right)$$

at 5 seconds:

$$Q(5) = C\mathcal{E}\left(e^{-5/RC}\right) = 5.4 \times 10^{-33}\mathrm{C}$$

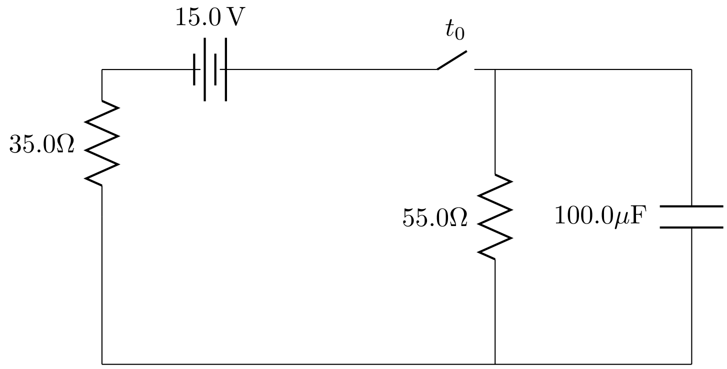
at 50 seconds:

$$Q(50) = C\mathcal{E}\left(e^{-50/RC}\right) = 4.3 \times 10^{-297}\mathrm{C}$$

What about 50,000 seconds?

$$Q(50000) = C\mathcal{E}\left(e^{-50000/RC}\right) = 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,00\mathrm{C}$$

3. In the circuit below, find the current passing through each branch and the charge on the capacitor at the following times: once the switch is closed, and a long, long time later.



Solution: First start with the loop rule when the switch is first closed and there is zero charge on the capacitor:

$$15.0\text{V} - I35.0\Omega - \frac{0}{100.0\mu\text{F}} = 0$$

The trick here is knowing that the current will follow the path of least resistance, which in this case is the capacitor branch at $t = 0$. One could also apply the loop rule to the rightmost branch and find the current on the 55.0Ω branch is zero.

We solve for the largest-loop current to find:

$$I_0 = \frac{15.0\text{V}}{35.0\Omega} = 0.4\text{A}$$

After a very long time, the capacitor is fully charged and we simply have a resistor-in-series circuit with an equivalent resistance of $R_{eq} = 90.0\Omega$, and so the current passes only through the left-most loop and has the value:

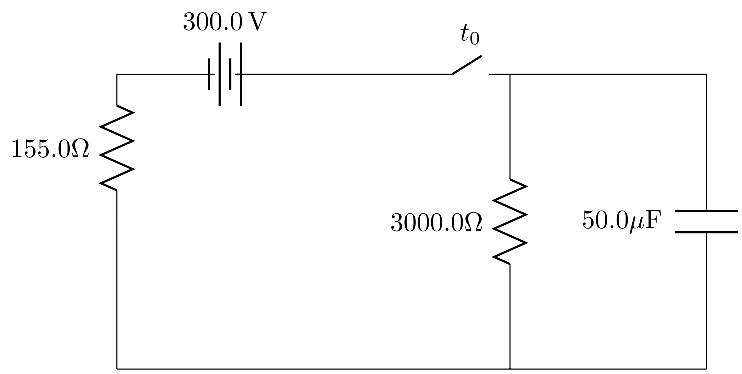
$$I_\infty = \frac{V}{R_{eq}} = \frac{15.0\text{V}}{90.0\Omega} = 0.2\text{A}$$

At this point the capacitor has maximum charge, and we can apply the loop rule on the rightmost loop to find that

$$0 = I_\infty(55.0\Omega) - \frac{Q_\infty}{100.0\mu\text{F}}$$

$$Q_\infty = 9.2 \times 10^{-4}\text{C}$$

4. In the circuit below, find the current passing through each branch and the charge on the capacitor at the following times: once the switch is closed, and a long, long time later.



Solution: First start with the loop rule when the switch is first closed and there is zero charge on the capacitor:

$$300.0\text{V} - I155.0\Omega - \frac{0}{50.0\mu\text{F}} = 0$$

The trick here is knowing that the current will follow the path of least resistance, which in this case is the capacitor branch at $t = 0$. One could also apply the loop rule to the rightmost branch and find the current on the 3000.0Ω branch is zero.

We solve for the largest-loop current to find:

$$I_0 = \frac{300.0\text{V}}{155.0\Omega} = 1.9\text{A}$$

After a very long time, the capacitor is fully charged and we simply have a resistor-in-series circuit with an equivalent resistance of $R_{eq} = 3155.0\Omega$, and so the current passes only through the left-most loop and has the value:

$$I_\infty = \frac{V}{R_{eq}} = \frac{300.0\text{V}}{3155.0\Omega} = 0.1\text{A}$$

At this point the capacitor has maximum charge, and we can apply the loop rule on the rightmost loop to find that

$$0 = I_\infty(3000.0\Omega) - \frac{Q_\infty}{50.0\mu\text{F}}$$

$$Q_\infty = 1.4 \times 10^{-2}\text{F}$$