Resistors and Circuits

Having introduced capacitors, we now expand our focus to another very important component of a circuit–resistors. This entails more interesting behavior in circuits.

Looking ahead: Our previous model of circuits assumed 100% efficiency. Why isn't that reasonable?

Students are expected to read the textbook in addition to homework and lectures. The lectures attempt to focus your attention on the key points of the topic, but the textbook offers rich details that can't be captured in allotted lecture time.

Lecture outline:

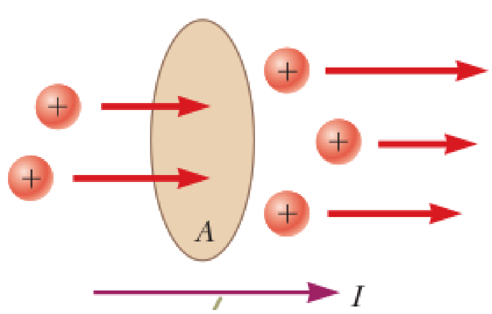
- The flow of charges: current
- Resistance: taxes
- R circuits
- RC circuits

1 Guiding Questions

What causes current to flow? How can it be stopped? How can it be stored? How can it be lost? What happens when we combine Resistors and Capacitors?

2 A bank account for electricity II: flows and taxes

We have talked about the movement of charges without explicitely discussing the movement of charges– such as when we described the charging of capacitors by a battery. Here, we will quantify the movement of charges in greater detail.



To talk about the movement of charges, we need to have a frame of reference. For example, if you stand on the ground and a car rides by at 10 mph, then it appears to you that the car is moving by at 10mph. Now suppose you are on a road moving 20 mph and you pass a car heading in the same direction at 10 mph. How fast will that car appear to be going? -10 mph.

1. Explain why the car will appear to go different speeds to different observers.

Solution:

For curcuits, our frame is the circuit itself–that is, we should be in the rest frame of the circuit. his point may seem trivial, but it will matter later in this course.

From the image in the previous page, we can measure the amount of charge passing through the cross-sectional area for a period of time, in which case we define the *average* **current** as being:

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

As you probably suspect, we can take this to infinitesimal limits with Calculus and define the *instantaneous* **current** to be,

$$I = \frac{dQ}{dt}$$

2. The distribution of a particular set of charges through a certain position in space is given by, $q(t) = (3t^2 - 2t)$ C. Find the current at this point as a function of time:

Solution:

$$I = \frac{d}{dt}q(t) = (6t - 2) \quad \text{C/s}$$

The units on the current are C/s, or as you might guess, we use a new unit named A for Ampere.

2.1 Drift current–transaction fees

If you think of charge as money, for a simplistic analogy, then the flow of money out of and into your bank account would be the equivalent analogy of current. Many folks have their bank account tied to a debit card or even a credit card. In terms of an ideal circuit, current flows without any loss of energy; the monetary analogy would be the flow money into or out of an account without any transaction fees. In the real world, this is not the case for circuits–energy is loss due to an intrinsic quality of the wires/medium through which current flows call resistance. Just as, for example, every purchase you may make on a credit card will cost you a certain fraction of that transaction, r, so too when a current passes through a wire, a certain amount of energy is loss due to resistance R.

The monetary analogy is only meant to help you frame this discussion in the context of a real-world example we are all familiar with. Let's now put this discussion on sold physics ground.

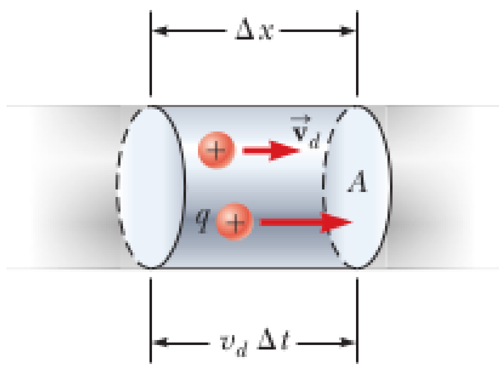
Most of the time we will be discussing current, we will do so in the context of circuits. So now is a good time to ask, what does it mean *physically* for charge to flow through a wire?

We can get a good approximation for the physics behind flowing current in a wire by making some simplifying assumptions.

3. Refresher question: Which of the following defines capacitance?

   ○ $C = QV$
   ○ $C = Q/R$
   √ $C = Q/V$
   ○ $C = V/C$

Let's look at a cross section of a wire:



The segment has caps with areas of A and the segment has length Δx. Let n represent the number of charges per unit volume, e.g. if there were three charges (all same value) in a volume of one cubic meter, $n = 3/m^3$. Each charge carrier has a charge of q. The total charge in this segment is;

$$\Delta Q = (nA\Delta x)\,q$$

If these charges are in motion with a velocity v_d, then the amount of time they take to pass through the segment is $\Delta x/v_d = \Delta t$. In a more realistic picture, the charges will have a distribution of velocities where the average velocity is v_d, and the d subscript means **drift**. This name implies, correctly, that the charges don't march along in a straight line, but rather have a randomness in their motion such that their net motion is in the direction of the current.

We can thus replace Δx with $v_d\Delta t$, and,

$$I_{avg} = \frac{\Delta Q}{\Delta t} = \frac{nAv_d\Delta tq}{\Delta t} = nqv_dA$$

It is physically reasonable that the current should be proportional to the area of the cross-section of the wire.

4. What is the charge on a wire that has a cross-section area of 0.00004m^3 with electrons drifting along with a drift velocity $v_d = 0.003\text{m/s}$ and the density of the charges in the wire is $5.3 \times 10^{28}/\text{m}^3$. The charge of the electrons is, -1.602×10^{-19} C. What is the current?

Solution:

$$I = 5.3 \times 10^{28}/\text{m}^3(-1.602 \times 10^{-19}\text{C})(0.003\text{m/s})(0.00004\text{m}^2) = -1.02 \times 10^3\text{A}$$

5. (This question is a lecture demo, will not be this involved on test.) Suppose a copper wire has a cross-sectional area of 3.31×10^{-6} (this is called 12-gauge wire by electricians) and carrying a current of 1.0 A. The molar mass of copper is 0.0635 kg/mol and Avogadro's number is $N_A = 6.02 \times 10^{23}$ mol^{-1}. The density of copper is 8940kg/m^3. What is the drift speed of electrons (charge -1.602×10^{-19} C)?

Solution: Assume, roughly, that each atom of copper contributes one electron to the flow of current (think of a relay race where each atom passes an electron to the one to the right of it, receiving one in turn from the one to the left of it; a more complicated but realistic model has that direction randomized but with a net drift in the direction of the current).

First we need to find the density of charges:

$$n = \frac{N_A}{V}$$

Since we know the density of copper, $\rho = M/V = (0.0635\text{kg/mol})/V$, then we have,

$$n = \frac{N_A \rho}{(0.0635\text{kg/mole})} = 8.5 \times 10^{28}\frac{1}{\text{m}^3}$$

So we have,

$$I_{avg} = nqv_dA \;\rightarrow\; v_d = \frac{I_{avg}}{nqA} = -2.2 \times 10^{-5}\text{m/s}$$

So slow? *Its the electric field that travels at the speed of light, not the charges themselves.* **This is an important point and you will most definitely be quized on this point!**. The wire in circuits serves as a medium for this electric field. Also, what does the negative sign mean? The direction of a current is positive in the direction that positive charges travel *or* the opposite direction that negative charges flow. *This is another key point!* Electrons flow in the opposite direction of the current (only because of convention).

2.2 Taxes

We looked at examples in our last class about, for example, a proton in a uniform electric field. Recall that the proton *accelerated* in the field. Now think about electrons in the electric field that is uniformly distributed through a wire (by hooking it up to a battery, for example). They too will accelerate, and in an ideal scenario, the accelerate forever and eventually reach infinite speed–infinite current. This is obviously not the case. In the real world, the current in a wire is **proportional** to the voltage of the battery and **inversely proportional** to a property of the wire called **resistance**:

$$I = \frac{\Delta V}{R}$$

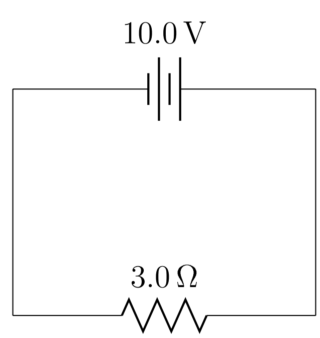
The higher the resistance, the lower the resulting current. The lower the resistance, the higher the resulting current. Resistance is defined by $R = \Delta V/I$ and has units $1\Omega = 1V/A$ where the Greek symbol Ω stands for the unit **ohm**.

Your text will have more details on resistance, and how, for example, it is related to the size of a wire, not just the material of a wire. Due to the loss of class time on Monday, we are not going to explore that in great detail. Please read your textbook on this topic. We will proceed to focus only on circuits.

To bring our financial analogy to a point of closure, resistance could be seen as taxes on capital gains that limit the amount of money that flows between assets. Since that is an analogy that doesn't really work so well, this brings to a close our use of money as an analogy for the flow of charges.

3 Resistors in circuits

Instead of considering the nature of the wire, its cross-sectional area and its length and so on, in order to calculate the resistance in the wire, we represent the resistance as a single piece of the circuit called a **resistor**:



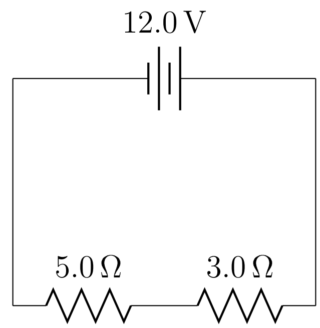
The jagged line represents resistor.

6. What is the current in the circuit shown above?

Solution:
$$I = \frac{V}{R} = \frac{10.0\text{V}}{3.0\Omega} = 3.3\text{A}$$

3.1 Resistors in series

We previously talked about capacitors in series and parallel, and now we need to have this same discussion with resistors. Consider the following circuit.



Our previous reasoning in regard to capacitors was based on the physics of charges and shared voltages, but to proceed we can introduce a new set of rules based on conservation of energy. They are:

- **Junction Rule:** At any point on a circuit, the current entering that point *must equal* the current exiting that point. For example, if two branches join to form one branch, the current in the single branch is the sum of the currents in the original two branches.
- **Loop Rule:** The voltages drops ΔV across each part of a circuit, as we circle around a circuit, adds up to zero. Going in the direction of current, batteries introduce a positive change in voltage, resistors a negative change in voltage ($\Delta V = IR$) and capacitors also introduce a negative change in voltage $\Delta V = Q/C$. By going in the opposite direction of current, those signs would be reversed.

Looking back at our new circuit, we can see that the two resistors **must** share the same current–current is conserved and it can't disappear. Let's follow the loop rule on this circuit:

$$0 = \Delta V_{batt} - IR_1 - IR_2 = 12.0\text{V} - I(R_1 + R_2)$$

This means that for resistors connected in series, the equivalent resistance–the one resistor that we could use to replace all of the resistors, is equal to the sum of all resistors in the original circuit:

$$R_{eq} = R_1 + R_2 + \cdots$$

7. What is the equivalent resistor in the above circuit?

Solution:

$$R_{eq} = 5.0\Omega + 3.0\Omega = 8.0\Omega$$

What about when the circuit is in parallel? In this case, the resistors in parallel share the same voltage, but **not the same current** (except in the case when each branch has the exact same resistance). In this case the *junction rule* applies, where we have (for a two branch circuit):

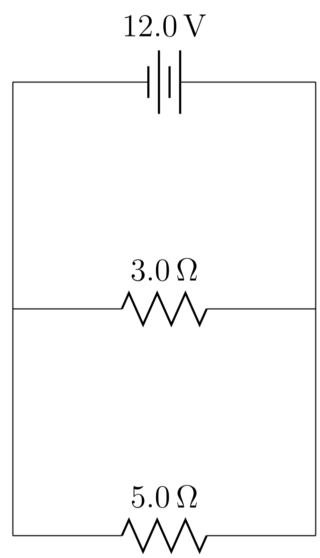
$$I = I_1 + I_2$$

Now our goal is to find an equivalent circuit which will only have the current I, so that

$$I_{eq} = \frac{\Delta V_{batt}}{R_{eq}} = \frac{\Delta V_{batt}}{R_1} + \frac{\Delta V_{batt}}{R_2}$$

And so the general formula for the equivalent resistor in a parallel circuit is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$



8. What is the equivalent resistor in the above circuit?

Solution:

$$R_{eq} = \frac{1}{\frac{1}{5.0\Omega} + \frac{1}{3.0\Omega}} = 1.9\Omega$$