Capacitance

We've so far talked about electric charges, the fields they create, and electric potential as a useful concept to help us figure out the interactions two charged particles will have. All of this is interesting unto itself, but we as engineers can't help but ask ourselves, *OK, what can I do with this?*. It is a fantastic question, and we will see in detail many things that can be done by knowing electromagnetism works. Before we can do something with electricity, we need some way to control it. This lecture talks about capacitance, the storage of electrical energy.

Looking ahead: When a capacitor stores up electrical energy, it does so over a period of time rather than instantly. In today's lecture, that fact will not be made use of, but in future lectures, we will find that this feature of capacitors leads to incredibly useful behaviors in circuits.

Students are expected to read the textbook in addition to homework and lectures. The lectures attempt to focus your attention on the key points of the topic, but the textbook offers rich details that can't be captured in allotted lecture time.

Lecture outline:

- A bank account for electricity
- Potential energy (potential spending)
- Dielectrics (earned interest on your bank account)
- Further notes

Guiding Questions

How can we control the flow of electricity? How does the analogy between a river dam and a capacitor work, and where does it not work? How do different materials store electricity in different ways?

A bank account for electricity

Picture this, you are watering a garden with a hose on full blast. Some prankster comes around and with a big pair of garden hedges and cuts your hose near the source, holding up his end while you hold up your end. What happens to the water in the hose? It stays in the hose, of course, until one of you drop their end of the hose.

We can imagine the same concept in with electricity. If we charge up an object (think for example the fur and rod example shown in class) but then isolate it, that charge that we put on the object will stay there until we discharge it in some way.

The potential V of that charged object due to the total charge Q_{tot} on it depends on the size/shape of the conductor as well as the total charge–a long thin rod will have a different series of equipotentials than will a sphere. (As per last class, we follow the convention of setting the potential to be zero at infinity.) If two charged objects (assume equal charge with opposite sign) are near each other, the potential difference between them, V, is of interest. It is the ratio of total charge to potential difference that we define as capacitance. An isolated conductor has **self-capacitance**, and a **capacitor** are two conductors near each other which have a potential difference V.

1. Which of the following defines capacitance?

   ◯ $C = QV$
   ◯ $C = Q/R$
   ◯ $C = Q/V$
   ◯ $C = V/C$

2. The unit for capacitance is called a **farad** F (*not* to be confused with Fahrenheit which we don't use in this class). What is a farad in terms of units we've already learned?

3. Why and how did Benjamin Franklin try to murder a turkey?

Please read the text book on the various types of capacitors, including the spherical capacitor and the Leyden jar. However, going forward we are going to focus exclusively on the parallel-plate capacitor (except on the homework).

0.1 Parallel-plate capacitor

4. Use Gauss' law to find the electric field between two parallel plates, and then use that finding to find the potential difference between those plates, and *then* use *that* finding to find the capacitance of two parallel plates. Assume charge distribution on the plate is σ and distance between them is d.

   This gives us the very interesting result that the capacitance of a parallel plate is dependent on the geometry of the plates alone, a result that is a fairly good approximation in the real-world for a wide range of applications, but which breaks down terribly on the smaller scales.

5. Why would this finding break down on smaller scales?

6. Find the capacitance of a parallel-plate where the plates are thin squares 10 cm on each side separate and are separated by 1.00 mm. If the plates are connected via a 12V batter, how much charge is transferred from one plate to another?

7. Show that the energy stored in a capacitor is $U = \frac{1}{2}CV^2$,

8. Find a parallel-plate capacitor configuration which, when hooked up to a 12 Volt battery, results in stored energy of $6.2 \times 10^{-9} J$. What happens to that energy when the battery is disconnected and the distance you selected is doubled? Show that this change in energy is equal to the work you did on the plate to move it.

Recall that we found that the capacitance of a parallel-plate is dependent on its geometry. Using that finding, we also find that,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 E^2 (Ad)$$

We define the energy density to be the energy per unit volume, i.e.,

$$u_e = \frac{1}{2}\epsilon_0 E^2$$

So one great way to look at where the money is stored is to think of it as being stored in the electric field, just as your bank doesn't really store your money in a vault at your local branch, but rather in a sort of monetary cloud.

9. Derive the equations that describe what happens when capacitors are wired up in series, and when they are wired up in parallel.

10. Define a dielectric, and how is it like a bank account that pays you an interest rate? How is it not like such a bank account?

11. What formula that describes the energy stored in a parallel plate capacitor that has a dielectric.

12. Consider two open-air parallel-plate capacitors hooked up to a 12V battery in series. Both capacitors have capacitance of $2\mu F$. One has Mica ($\kappa = 5.4$) inserted between the plates. How does the charge configuration change? Repeat this but now instead have them wired in parallel.