Electric Potential

Gravitational Potential was discussed in the context of Energy and Mechanics in Physics I. When we talk of Electric Potential, we are **not** talking about energy, but energy per unit charge. **Electrostatic Potential Energy**, which is the electric potential function multiplied by a charge, is analogous to gravitational potential energy, and so the powerful concepts of energy conservation are useful here as well.

Looking ahead: Given that gravity and electrostatic force have very similar inverse-r-squared forms, it would seem natural that we can use some ideas from the concept of gravitational potential for electrostatics, but be careful in being too eager to make the analogy–since gravity is only attractive while electric charges can be repulsive or attractive depending on the charge combination, we need to always keep their differences in mind.

Lecture outline:

- Defining Potential Difference
- Relating this to Energy
- A conservation of Energy Example
- Equipotentials

Guiding Question

How much energy does it take to ionize a hydrogen atom–that is, how much energy do we need to input into a hydrogen atom in order to completely liberate the electron from the proton? We will be able to answer this question at the end of the lecture.

Conservative Forces and Work

1. Why is the electric force conservative?

   ◯ The path taken alters the energy such that the final energy of a system directly depends on where it went.
   ◯ It isn't
   ◯ The final energy state is only dependent on the final position of an object in the system.

2. Which is an example of a non-conservative force?

   ◯ Spring Force ◯ Gravitational Force ◯ Friction Force

   The defining nature of a conservative force is that the total energy of the system isn't dissipated, though it may be transferred between different types of energy–it still remains **within the system**. For example, a roller coaster's kinetic energy is converted into gravitational potential energy as it climbs a hill, but the total energy of the roller coaster remains the same (as long as we ignore friction).

   A good analogy would be if you had a savings and checking account tied together. Transferring 100$fromyoursavingstoyourcheckingdoesn'tmeanthatthe$100 disappears–your total net worth stays the same until you spend that money. Think of conservative force in terms of transferring money between a savings and a checking account.

   On the other hand, non-conservative forces are those in which the energy of the system is leaked off into the rest of the universe in some way. Think of a book sliding across a table–it eventually comes to a stop because friction has taken the books energy and converted it into thermal energy–heat–so that while that energy isn't destroyed, it is removed from the book. Think of friction as the bank fee that your bank may charge you just to have a checking account–they take money out and you will never get it back, so your total net worth isn't conserved.

   For conservative forces, the change in potential energy is given (in one dimension here) as,

   $$dU = -\vec{F} \cdot d\vec{x}$$

   For electrostatics we can write this as,

   $$dU = -q\vec{E} \cdot d\vec{x}$$

   since $F = qE$. We now define the **potential difference** function as

   $$dV = \frac{dU}{q_0} = -\vec{E} \cdot d\vec{x}$$

   $$\Delta V = -\int_a^b \vec{E} \cdot d\vec{x}$$

3. What are the units of the potential difference?

   ◯ N/C ◯ J/C ◯ $N \cdot C$

   We define the units to be called a V for volt. It is unfortunate that we use the same symbol, V, to talk about the electric potential itself; however, over a century of convention means we will just have to be careful not to confuse the two.

   Also note that you may, more often than not, see the electric field defined in terms of the V/m rather than in terms of N/C, but you can convince yourself that these two are equal.

The **electron volt** (eV) is a very common way of measuring energy in particle physics. It is defined as the energy of one electron under a 1 V potential,

$$1eV = 1.60 \times 10^{-19} C \cdot V = 1.60 \times 10^{-19} J$$

4. In which direction does the electric potential point? (trick question!)

0.1 Potential due to a point charge

So far we've worked with point charges, and for the most part, with an exception for Gauss's Law, your first quiz will focus on these as well. Recall that we found in our last lecture that

$$\vec{E}_p = \frac{kq}{r^2}\hat{r}$$

So that now we can directly integrate:

$$\Delta V = V_f - V_i = -\int_{r_i}^{r_f} \frac{kq}{r^2}\hat{r} \cdot d\vec{r} = -\int_{r_i}^{r_f} \frac{kq}{r^2} dr = \frac{kq}{r_f} - \frac{kq}{r_i}$$

Since the electric field goes as $1/r^2$, it decreases very rapidly the further away we are from the point particle, and so we tend to treat r_i as being so far away that we can treat it as infinity ($r_i \to \infty$) and we simply write r_f as r. Then, the electric potential to take a test charge from far far away (infinity) to a position r away from another charge q is

$$V = \frac{kq}{r}$$

This is conventionally called the *Coulomb Potential*, and

$$U = q' \frac{kq}{r}$$

is called the *Electrostatic Potential Energy of a Two-Charge System.*

5. What is the electric potential at a distance $r_0 = 0.529 \times 10^{-10}$ m from a proton? This is the distance of an electron away from a proton (approximately, and for ground-state only).

6. Here's the big problem–how much energy would it take, in terms of eV, to liberate an electron from a hydrogen atom. *Hint: Recall that centripital acceleration is given as* $F_c = mv^2/R$.

Conservation of Energy

7. Imagine a universe where there exists only a proton and an electron. The electron is located at the center of the universe and the proton is located an infinite distance away and released from rest. Assume that in this universe, the central electron is held in place somehow (magic?). How fast will the proton be traveling when it is one meters away from the electron?

   Hint: We can also define the electrostatic potential energy of a system as the work needed to bring the charges from an infinite separation to their final positions. Use the kinetic energy - work theorem to solve from there, i.e. $W_{net} = \Delta KE$