Electric Fields & Gauss's Law

We discovered in part one of this series that there are two main classes of forces. There are **contact forces** which require bodies to be in physical conduct, and there are **action-at-a-distance forces** (also called **field forces**) which act without physical contact.

Looking ahead: It may seem almost magical that particles separated by distances can somehow exert forces on one another. We now know precisely how this is done–by an exchange of certain types of particles. In the case of electricity, the particle that causes attraction is called a *virtual photon*, and when we learn about electro-magnetic waves, we will learn about photons. Photons are what makes up light as well, so the very light by which you are reading this is related to this topic.

Lecture outline:

- Electric charges
- Induction
- Coulomb's Law
- Electric Field
- Electric Flux
- Gauss's Law
- Application of Gauss's Law

Electric Charge and Induction

1. Which of the following is not a field force?

   ◯ Gravity ◯ Normal Force ◯ Electricity ◯ Magnetism ◯ Nuclear Forces

2. What is an electrical charge?

3. Who first proved that charge is quantized?

   ◯ Newton ◯ Einstein ◯ Millikan ◯ Feynman ◯ Gilmore

4. Which of the following is not a charge carrier? Also, put the charge symbol next to the ones that are carriers (+ or −). ◯ Electron ◯ Proton ◯ Neutron

5. Who named the charges *positive* and *negative*? Why did this person not name them as best they could have?

6. What is the most very fundamental charge carrier?

7. What is the difference between conductors and insulators?

8. What is induction, and is that the same as acquiring a shock form shuffling your feet on a carpet?

9. Why is the earth a ground?

Coulomb's Law

You may have learned in the first part of this course (though if you didn't, you will learn in the next part) that the gravitational force is not as simple as mg (it is only near the surface of Earth) but is instead given by

$$F_g = \frac{Gm_1m_2}{r^2}$$

where G is the universal gravitaional constant, m_1 and m_2 are the masses of two bodies attracting, and r is their separation.

It turns out that electrical point charges have a similar "inverse square law", where the force of an electron is given as,

$$F_e = k_e\frac{|q_1||q_2|}{r^2}$$

where

$$k_e = 8.9876 \times 10^9 \frac{\text{N m}^2}{C^2}$$

another way we can write this, which will make more sense next week, is

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where ϵ_0 is called the permittivity of free space,

$$\epsilon_0 = 8.8542 \times 10^{-12}\text{C}^2/\text{N m}^2$$

10. The SI unit of charge is called a
    ◯ Newton N ◯ electron e ◯ Kelvin K ◯ Coulomb C

11. What is the smallest unit of *free* charge? The smallest unit of charge?

12. (See figure drawn on board as well). Two positively charged particles fixed in place on an x-axis have charges $q_1 = 1.60 \times 10^{-19}C$ and $q_2 = 3.20 \times 10^{-19}C$ with separation $r = 0.0200m$. What is the magnitude and direction of the force F_{12} on particle 1 from particle 2?

13. Now let's make the previous problem a little more complicated. Add a new charge $q_3 = -3.20 \times 10^{-19}C$ located (3/4)r away from particle one at an angle of 60°. What is the net force on particle 1 due to the charges q_2 and q_3?

Electric Field

The concept of a field is a mathematical abstraction of the idea that certain things can be measured at every point in space and have differing values. If you measure the temperate for each square mile of the United States, for example, the resulting distribution of temperatures (measurement plus coordinates) is a temperature field.

Temperature is a scalar, while electrical force is a vector. Thus when we measure electricity, we measure which direction is more negative or positive than the others. Another example of a vector field is with magnets, which we will cover later. If you travel around the world and look at your compass, it will point towards magnetic North (more or less) with varying strength. If you had a GPS with you and recorded your position on Earth, that information (field direction and strength plus position) is a field.

How can we measure the electrical field? We can place a test charge q_0 at a position, and see in which direction and with what force it is pulled. We define the electric field at that point as being:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

14. What are the SI units for the electric field?

○ N/C ○ C/m^2 ○ ϵ_0 N/m^2

Of course, this makes it easy to relate the force that an arbitrary charge experiences inside an electrical field as being

$$\vec{F}_e = q\vec{E}$$

Why are fields so useful? Well, a great many charges are point charges, which means that the corresponding force they produce is based directly on Coulomb's Law. Let

$$\vec{F}_e = k_e \frac{q q_0}{r^2} \hat{r}$$

for the before mentioned test particle, where q is the charge of some point particle creating an electrical field. Then the field is given as

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

15. Draw the field lines for a positive and negative charge close to each other.

16. Return to problem 13. Now, charge q_2 and q_3 are fixed in place somehow. What is the acceleration on charge q_1? Remember Newton's Second Law (F = ma).

In general, the electrical field can be given via Calculus as

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r}$$

so that, for example, the electric field due to a uniformly charged disk with radius R and surface charge density (charge per surface area) σ is given along the axis of the disk as

$$E = k_e x \pi \sigma$$

Gauss's Law

Physics tries to find simple ways to handle complex problems. It is often the case that we are not given a simple system with a single point charge. Instead, we might have multiple point charges or charges spread out along a cylinder (as in a wire). It can sometimes be easier to relate the electric field on an imaginary surfaces surrounding charge to the charge that this surface encloses.

Flux is the rate of flow through an area, or a volume flow rate. For example, the flux through a water pipe is equal to the velocity of the water times the cross-sectional area of the pipe. That is, we can define flux as being $\Phi = \vec{v} \cdot \vec{A}$ where we call this the dot-product. More specifically, $\Phi = vA\cos(\theta)$, where θ is the angle difference between the axis of the area in question and the velocity vector.

For electric fields, we define the **electric flux** as

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

where the $\oint$ means we are integrating over the entire surface.

17. What is the Electric Flux of a cylinder of radius R placed in a uniform electric field with magnitude E?

Gauss's Law: Point Charge Version

The net flux through any closed surface surrounding a point charge q is given by q/ϵ_0 and is independent of the shape of that surface.

Proof: If the next flux is given as

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

then for a point charge, using Coulomb's Law, this is:

$$\Phi_E = \oint \frac{k_e q}{r^2} d\vec{A}$$

The surface area of a sphere is $A = 4\pi r^2$ and since the electric field is uniform over the surface of a sphere centered on the charge, then the flux is

$$\Phi_E = \frac{k_e q}{r^2} 4\pi r^2 = 4\pi k_e q$$

but recall that

$$k_e = \frac{1}{4\pi\epsilon_0}$$

so that

$$\Phi_E = \frac{q}{\epsilon_0}$$

Gauss's Law: General Version

The net electric flux of a surface surrounding an enclosed charge of any configuration is given as:

$$\Phi_E = \oint \vec{E} \cdot \vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

This law is useful when,

- The field has some symmetry (sphere, rod, disk, etc.)
- An appropriately simple Gaussian shell can be formulated (a sphere for a sphere, a cylinder for a rod, and so on).
- The field is such that the dot product of the field and the area of the Gaussian shell cancels in some convenient way.
- The electric field has some constant known value for the entire surface or part of the surface.

18. Find the electric field of a large nonconducting sheet with surface charge σ. Extend this to two conducting sheets brought parallel and near to each other.

19. In one early vision of the hydrogen atom, the electron orbited the proton like the moon orbits Earth. What is the net flux for a Gaussian sphere whose radius is halfway between the electron and proton? What is the electric field (without using Coulomb's Law)? What is the net flux of such a sphere whose radius extends beyond the electron? *This effect is sometimes said that the electron is shielding the proton.*

For spherical shells, we have the following important points:

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of a shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

Further examples as time permits.