Name: ____________________________

> Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

1. (6 points) Charge A is a -3.0μC point charge is placed at position $(x, y) = (2.0, 0.0)$ m, and charge B is a point charge $q_2 = 4.5$ μC is located at $(x, y) = (0.0, 1.0)$m.

   (a) If the electric potential is taken to be zero at infinity, find the total electric potential due to these charges at a point located at a point C at $(x, y) = (1.0, 1.0)$ m?

   Solution: We solve this by using the principle of superposition. That is, we find the potential at point C by finding the potential at that point due to charge A alone, then finding the potential at that point due to charge B alone, and then simply adding them up. Because we are taking the potential to be zero at infinity, we can use the equation that the potential due to a point charge q that is a distance r away from that point charge (remember that potential is **not** a vector so the direction doesn't matter in this calculation, just the sign),

   $$V = \frac{kq}{r}$$

   where $k = \left(8.987 \times 10^9 \frac{\mathrm{N}m^2}{C^2}\right)$ is a universal constant. The potential at point C due to the charge a point A is:

   $$V_{AC} = \left(8.987 \times 10^9 \frac{\mathrm{N}m^2}{C^2}\right) \frac{-3.0 \times 10^{-6}\mathrm{C}}{1.4\mathrm{m}} = -19064.3\mathrm{V}$$

   where we found the radius $r = \sqrt{(A_x - C_x)^2 + (A_y - C_y)^2} = 1.4$m, which is how we will find the radius in all cases.
   Next we find,

   $$V_{BC} = \left(8.987 \times 10^9 \frac{\mathrm{N}m^2}{C^2}\right) \frac{4.5 \times 10^{-6}\mathrm{C}}{1.0\mathrm{m}} = 40441.5\mathrm{V}$$

   Thus the total potential is:

   $$V_{tot} = V_{AC} + V_{BC} = 21377.2\mathrm{V}$$

   (b) How much work is required to bring a third point charge of 4.0 μC from infinity to the point P? Assume the mass is 2.0×10^{-10}kg.

   Solution:
   To bring a point in from infinity to the position P requires the work:

   $$W = q_C V_{tot} = 8.6 \times 10^{-2}\mathrm{J}$$

   (c) If you found the net potential to be positive, and if the third particle is postive, it will not want to go to that point and work needs to be done to put it there. In general, if the net potential and the third charge have the same sign, that third charge will want to go away from that point rather than toward it.

   On the other hand, if the net potential and the charge have opposite signs, then it would want to go towards that point.

If they have the same signs and the particle starts from infinity, then to bring that particle to point C, the electric force be doing negative work, i.e. work against that motion.

If they have opposite signs, then the electric force itself is doing positive work and the charge will accelerate naturally to that point.

Look at your numbers and figure out which case you have. If you have the first case, where the sign of the particle is the same as the sign of the net potential, assume that the third particle starts out at infinity with a velocity v pointing towards the point C. It so happens that this velocity is large enough that there is enough kinetic energy for the third particle to arrive at point C from infinity, slowing along the way, and coming to a complete stop at point C before it starts going backwards. Find v.

If you have the case where the third charge and the net potential have opposite signs, then assume the point particle starts out at infinty with zero velocity. How fast will it be going by the time it makes it to point C?

Solution: In either case, the work-energy theorem (or the conservation of energy if you prefer to think of the potential energy verses kinetic energy) gives:

$$\frac{1}{2}mv^2 = qV \quad \rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

And this gives us:

$$v = 29241.9\text{m/s}$$

2. (9 points) A proton is released from rest in a uniform electric field of magnitude -4×10^3V/m (determine from the sign which direction the field points in). The proton undergoes a displacement of 0.30 m.

(a) In which direction will the proton go when released? Why?

Solution: Since $F = qE$ and the proton has positive charge, it will travel in the direction of the electric field.

(b) Find the change in electric potential of the proton as a result of this displacement. *Hint: The electric field is uniform, and your textbook will be very helpful here.*

Solution: The physical rule is

$$\Delta V = -\int_i^f \mathbf{E} \cdot d\mathbf{l}$$

When the electric field is constant, finding the change in potential is

$$\Delta V = -Ed$$

where d is the displacement. In this case,

$$\Delta V = V_f - V_i = -(-4e3)(-0.30)\text{V} = -\left(1.2 \times 10^3\text{V}\right)$$

This means that it went to a lower potential.

(c) Find the change in electrical potential energy of the proton for this displacement and explain the physical meaning of the sign of the change. *Hint: Think of how the gravitational potential energy changes for an object sliding down a hill.*

Solution: The rule is that,

$$\Delta U = q\Delta V$$

The proton is positive, so ΔU will have the same sign as ΔV. The charge of a proton is 1.602×10^{-19} C, and so,

$$\Delta U = 1.602 \times 10^{-19}\ \text{C}\left(-\left[1.2 \times 10^3\text{V}\right]\right) = -1.9 \times 10^{-16}\text{J}$$

Think about this in terms of gravity. What does it mean when a particle goes to a lower gravitational potential energy? It means it "wants" to go as close to Earth as possible. An object in a field going where it would "rather" go is losing potential energy. Potential energy is, essentially, the potential kinetic energy the object could have if it was allowed to go to where it would "rather" be.

(d) Find the speed of the proton after it has moved 0.50 m starting from rest?

Solution: The mass of the proton is 1.672×10^{-27} kg. We require that energy is conserved, i.e.,

$$\Delta E = 0 = \Delta U + \Delta K \quad \rightarrow \quad \Delta K = -\Delta U$$

Then we have,

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\Delta U$$

Since it started from rest then $v_i = 0$, and we have (BE SURE TO RECALCULATE PREVIOUS PARTS USING D = -0.5),

$$\frac{1}{2}mv_f^2 = -\Delta U \quad v_f = \sqrt{\frac{-2\Delta U}{m}} = 6.2 \times 10^5 \text{m/s}$$