Name: ______________________________

You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

Multiple choice

1. 1 point If one experiences a centipital or centrifugal force, that means that one is
   - ◯ Not accelerating
   - √ **In a non-inertial frame**
   - ◯ In an inertial frame
   - ◯ Going in a straight line

2. 1 point Why are high-way off-ramps banked at an angle?
   - ◯ To create a centriptial force.
   - ◯ To save money.
   - ◯ To slow drivers.
   - √ **To counterbalance the centrifugal force.**

3. 1 point Newton's Second Law does say that.
   - ◯ Net torque is equal to angular acceleration.
   - ◯ Net torque is equal to linear acceleration.
   - ◯ None of the torques acting on an object have reaction forces.
   - √ **The sum of all torques on an object is proportional to its angular acceleration.**

Short Problems

$$
\begin{aligned}
F_{\text{cent}} &= m\frac{v^2}{R} \\
a_c &= \frac{v^2}{R} \\
f_s &\leq \mu_s F_N \\
f_k &= \mu_k F_N \\
\sum \tau_{\text{net}} &= I\alpha \\
K_r = \frac{1}{2}I\omega^2 &\quad , \quad K_{linear} = \frac{1}{2}mv^2 \\
L &= I\omega
\end{aligned}
$$

1. $\boxed{\text{3 points}}$ Indiana Jones (no relation) has a mass of 85.0 kg and crosses a river by swinging across a vine. The vine is 10 meters long and assume he's at the very end of it. His speed at the very bottom of the swing is 8.00 m/s. The vine has a breaking strength of 1000 N, which he is unaware of. In the river are a large gang of hungry aligators. Will the aligators continue to be hungry (the vine is strong enough to carry Dr. Jones across the river) or will it snap and feed them?

Solution: This is a centripital force problem, where

$$F_c = m\frac{v^2}{R} = 85.0\frac{8^2}{10} = 544N$$

His weight is,

$$F_g = mg = 85(9.8) = 833N$$

Newton's Second Law gives either (for the frame of reference of the ground which is an inertia frame):

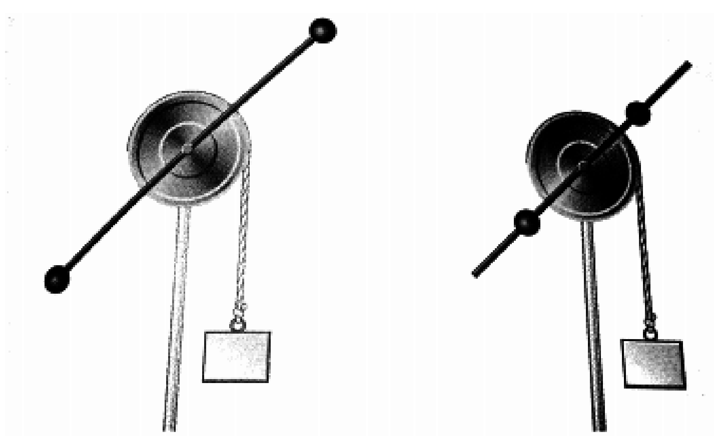
$$\sum F_r = T - F_g = F_c$$

or (in the frame of Indiana which is non-inertial and thus has fictitious forces)

$$\sum F_r = T - F_g - F_c = 0$$

The total tension is $T = F_g + F_r = 1377.0$ N, so the vine will snap.

2. $\boxed{\text{2 points}}$ Which of the following two systems (left or right) will spin the fastest and why? The solid disk, spheres, and box all have the exact same mass, the only difference is the location of the spheres which alters the inertia. *Hint: You can either apply the concept of angular momentum conservation or apply Newton's Second Law in the angular form, and recall that for spheres which can be treated as points,* $I = MR^2$.



Solution: The one on the right. Since they experience the same torque from the identical weights, the only difference is I, the moment of inertial. Since I is proportional to the distance of the masses from the center of rotation, I on the right is smaller. Thus

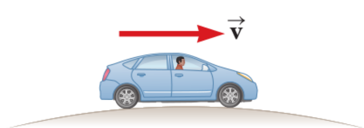
$$\sum \tau_{\text{net}} = F_g R = I\alpha$$

Since $F_g R$ doesn't change, but I is different for each, then α, the angular acceleration must also differ. For the smaller I, α will be larger. For the larger I, α will be smaller.

Longer Problems

$$\begin{aligned}
F_{\text{cent}} &= m\frac{v^2}{R} \\
a_c &= \frac{v^2}{R} \\
f_s &\leq \mu_s F_N \\
f_k &= \mu_k F_N \\
\sum \tau_{\text{net}} &= I\alpha \\
K_r = \frac{1}{2}I\omega^2 &\quad , \quad K_{linear} = \frac{1}{2}mv^2 \\
L &= I\omega \\
U_g &= mgh
\end{aligned}$$

1. $\boxed{\text{5 points}}$ Nobel laureate Arthur Holly Compton designed a speed bump to be installed outside of his workplace to slow down traffic. It is essentially a hill in the road. Suppose a 1800 kg car passes over the hump in the road which is an arc of a circle of radius $R = 20.4$ m. What is the maximum speed the car can have without losing contact with the road?



Solution:

Like the vine problem, we can do this in either the car's non-inertial frame, or the ground's (inertial) frame. Let's work in the ground's frame:

$$\sum F_y = F_N - F_g = F_c$$

Now, the condition to lose contact with the ground is that $F_N = 0$ since F_N is the contact force, in which case we have $-F_g = F_c$. The sign only means that they are opposite directions.

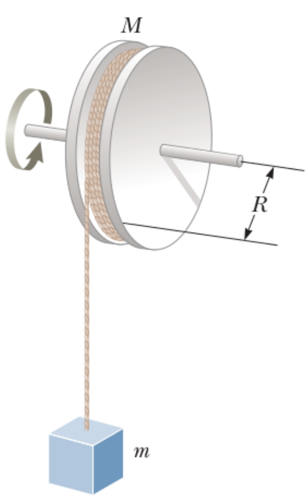
We have then:

$$-1800kg(-9.8\frac{\mathrm{m}}{\mathrm{s}^2}) = 1800kg\frac{v^2}{20.4\mathrm{m}} \quad \rightarrow \quad v = 14.1\frac{\mathrm{m}}{\mathrm{s}}$$

Or 31.5 mph.

2. $\boxed{\text{5 points}}$ An object with mass $m = 5.10$ kg is attached to an ideal rope which is wrapped around a reel of radius $R = 0.250$ m and mass $M = 3.00$ kg. The reel is a solid disk with $I = \frac{1}{2}MR^2$. The object is released from rest 6.00 m above the floor.

What is the speed of the object when it hits the floor? *Hint: assume non-slip conditions whereby* $v = \omega R$ *and use energy conservation concepts.*



Solution: This can be solved two different ways: by using Newton's Second Law or by using Conservation of Energy.

For conservation of energy, $\Delta U_g + \Delta K = 0$ so

$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}I\frac{v^2}{R^2} + \frac{1}{2}mv^2 = mgh$$

$$\frac{1}{4}Mv^2 + \frac{1}{2}mv^2 = mgh$$

$$\left(\frac{M}{4} + \frac{m}{2}\right) v^2 = mgh$$

$$v = \sqrt{\frac{mgh}{\left(\frac{M}{4} + \frac{m}{2}\right)}} = 9.53\frac{\mathrm{m}}{\mathrm{s}}$$

OR, solve with Newton's Second Law:

Use Newton's Second Law for the mass and the pulley:

$$\sum F_y = T - mg = -ma$$

$$\sum \tau_{\mathrm{net}} = TR = I\alpha = I\frac{a}{R}$$

The last one assumed the no-slip condition. Solve for $T = m(g-a)$ so $m(g-a)R = Ia/R$. Solve for a,

$$m(g-a)R^2 = Ia, \ \rightarrow \ mgR^2 = \left(I + mR^2\right) a \ \rightarrow \ a = \frac{mgR^2}{I + mR^2} = \frac{mgR^2}{0.5MR^2 + mR^2}$$

Simplify by dividing out the R^2 terms, so that $a = 49.98/6.6 = 7.57\frac{\mathrm{m}}{\mathrm{s}^2}$. Using kinematic equations, $x_f = x_i + v_i t + \frac{1}{2}at^2$ $6 = \frac{1}{2}(7.57)t^2$ gives $t = 1.26$ s and so $v_f = v_i + at = 0 + 7.57(1.26) = 9.53$ m/s.