Name: Student

You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

Short answers

1. 2 points When an object is immersed in a liquid at rest, why is the net force on the object in the horizontal direction equal to zero?

Solution: The pressure is the same on points that are at the same level but on opposite sides.

2. 2 points How would you determine the density of an irregularly shaped rock?

Solution: Weigh it in air then weigh it in water. The difference in those weights is due to buoyancy. That buoyancy tells you the weight of a volume of water displaced by the rock whose volume is the same as the rock. Since you know the density of water is $\rho_w = 1000\ \text{kg/m}^3$, $B = \rho_w g V$ and solve for V. Divide the weight you got for the rock in air by $9.8\ \frac{\text{m}}{\text{s}^2}$ to get the mass, and then divide the mass by the volume you figured out previously. This is the same as the golden crown problem we saw in class.

3. 2 points Can an object be in equilibrium if it is in motion? Explain.

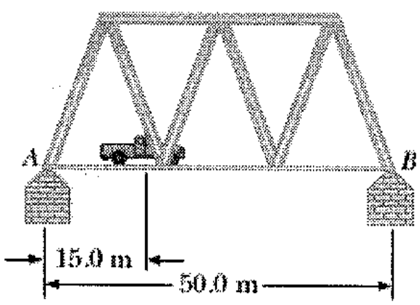
Solution: Yes, the condition of equilibrium is the all of the net forces and all of the net torques is equal to zero. If an object is in motion at a **constant speed** and **constant direction**, and/or is rotating at a **constant rate**, then that means there is no linear or angular acceleration which implies that the net force and net torque are equal to zero–which implies equilibrium. If the object has any acceleration then it is not in equilibrium.

4. 2 points (a) Give an example in which the net force acting on an object is zero and yet the net torque is nonzero. (b) Give an example in which the net torque acting on an object is zero and yet the net force is nonzero.

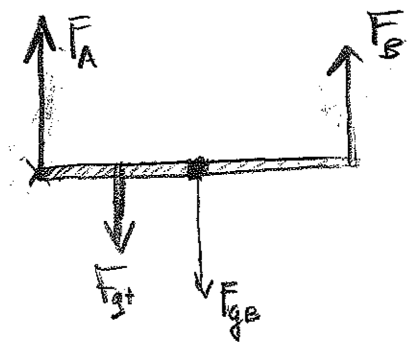
Solution: (a) A rod free to rotate about an axis that passes through its center has one force pushing down at its right tip and another force of the same magnitude pushing up on its left tip, resulting in a net torque but zero net force. (b) Consider the same rod when a force is pushing it from the right in a direction that is parallel to the rod. Because this force passes through the center of rotation, there is no net torque but certainly a net force.

Shorter problems

5. [4 points] A bridge of length 50.0 m and mass 8.00×10^4 kg is supported on a smooth pier at each end. A truck of mass 3.00×10^4 kg is located at 15 m from one end. What are the forces on the bridge at the points of support. Assume the metallic supports distributes the forces evenly along the bridge so that you don't need to include it in your equations.



Step 1



$F_{gt} = m_t g = (3E4)\text{kg}(9.8\ \text{m/s}^2) = 2.94 \times 10^5$ N

$F_{gB} = M_B g = (8E4\text{kg})(9.8\ \text{m/s}^2) = 7.84 \times 10^5$ N

Step 2

$\Sigma F_x = 0$

$\Sigma F_y = F_A + F_B - F_{gt} - F_{gB} = 0 \rightarrow \boxed{F_A + F_B = 10.78 \times 10^5\ N}$

$\Sigma \tau = 0(F_A) - 15\text{m}(F_{gt}) - 25\text{m}(F_{gB}) + 50\text{m}(F_B) = 0$

$4.41 \times 10^6\ [N \cdot m] + 19.6 \times 10^6 [N \cdot m] = 50\text{m}\ F_B \rightarrow \boxed{F_B = 4.8 \times 10^5 N}$

$\boxed{F_A = 5.98 \times 10^5 N}$

6. [4 points] A 200-kg load is hung on a wire of length 4.00 m, cross-sectional area $0.200 \times 10^{-4} m^2$, and Young's modulus 8.00×10^{10} N/m^2. What is its increase in length?

$$Y = \frac{F/A}{(\Delta L)/L} = 8.00 \times 10^{10}\ \text{N/m}^2 = \frac{[200\text{kg}(9.8\ \text{m/s}^2)]/[0.200 \times 10^{-4}\ m^2]}{\Delta L / 4.00\text{m}}$$

$\Delta L = 0.0049$ m

7. $\boxed{\text{4 points}}$ The deepest point in the ocean is the Mariana Trench (11 km). The pressure at this depth is 1.13×10^8 Pa (recall that 1 Pa = 1 pascal = N/m^2) Calculate the change in volume of 1.00 m^3 of seawater carried from the surface to this deepest point. Assume the surface pressure is 1 atm = 1.013×10^5 Pa. The Bulk modulus of water is 0.21×10^{10}. Is it a good approximation to think of water as being incompressible?

$$B = -V_0 \frac{\Delta P}{\Delta V} \longrightarrow \Delta V = -V_0 \frac{\Delta P}{B} = (-1.00\,m^3)\frac{[1.13\times10^8\,N/m^2 - 1.013\times10^5\,N/m^2]}{0.21\times10^{10}\,N/m^2}$$

$$\boxed{\Delta P = P_f - P_i = 1.13\times10^8\,Pa - 1.013\times10^5\,Pa}$$

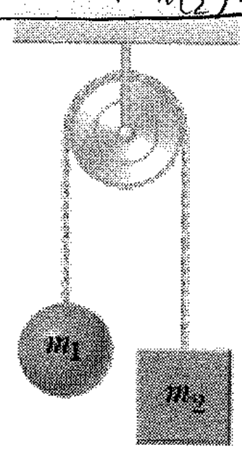
$$\Delta V = -0.0538\,m$$

Each side shrinks from 1m to 0.982 m, So treating water as incompressible is a good approximation.

8. $\boxed{\text{6 points}}$ A 2.00 m-long cylindrical steel wire with a cross-section diameter of 4.00 mm is placed over a light, frictionless pulley. An object of mass $m_1 = 5.00$ kg is hung from one end of the wire and an object of mass $m_2 = 3.00$ kg is hung from the other. By how much will the wire stretch when the objects are released and allowed to move freely? Young's Modulus for steel is $Y_s = 20 \times 10^{10}$ N/m^2.

Long Problem

$4.00\times10^{-3}\,m \rightarrow A = \pi\left(\frac{d}{2}\right)^2 = 1.26E{-5}\,m^2$



T, $m_1 g$ +, $m_2 g$

Since the pulley is being treated as ideal, We don't have to consider angular physical concepts.

a<0 on heavier side

$$m_1: \ \Sigma F_y = T - (5.0kg)(9.8\,m/s^2) = (-5.0kg)a$$

$$m_2: \ \Sigma F_y = T - (3.0kg)(9.8\,m/s^2) = +(3.0kg)a$$

accelerates upwards on lighter side

$T - 49 = -5a$,

$T - 29.4 = 3a$

$T = 3a + 29.4$

$3a + 29.4 - 49 = -5a$

$-19.6 = -8a \rightarrow a = 2.45$

$a = 2.45 \rightarrow$

$T - 49 = -5(2.45)$

$T = 36.75\,N$

$$Y_s = 20\times10^{10}\,N/m^2 = \frac{F/A}{\Delta L/L} = \frac{36.8/1.26E{-5}}{\Delta L/2}$$

$\Delta L = 2.92\times10^{-5}\,m$

Longer problems

9. $\boxed{\text{15 points}}$ If you drop a brass sphere into the ocean, will it sink all the way to the bottom? Since the volume of the sphere decreases with increasing pressure, then that means that as it shrinks, its displacing less volume. To make this a Physics I problem, let's ignore the fact that the density of water changes.

   (a) Recall that the pressure at a given depth underwater is equal to $P = P_0 + \rho_w g \Delta h$. Assume the brass sphere has a volume $V_0 = 0.5\ \mathrm{m}^3$ at sea-level where the pressure is $P_0 = 1atm = 1.013 \times 10^5$ Pa. Let $\Delta P = P - P_0 = \rho_w g \Delta h$. Using the Bulk Modulus of brass $B_b = 6.1 \times 10^{10}\ \mathrm{N/m}^2$ and the Bulk Modulus equation, find a formula that relates the volume of the brass sphere ($V = V_o + \Delta V$) as a function of depth Δh.

   $$B = -V_0 \frac{\Delta P}{\Delta V} \longrightarrow \Delta V = -V_0 \frac{\Delta P}{B} = (-0.5)\frac{(1000)(9.8)(y)}{6.1\times 10^{10}} = -8.03\times 10^{-8} y$$

   $$V = (0.5 - 8.03\times 10^{-8} y)\ m^3$$

   (b) Now recall that the Buoyancy force in water is $\mathbb{B} = \rho_w g V_{disp}$. Again, assume for simplicity that the density of water doesn't change and is $\rho_w = 1000\ \mathrm{kg/m}^3$. Given what you found in the previous part of this problem, find a formula that relates the Buoyancy force on the brass sphere as a function of depth Δh.

   $$\mathbb{B} = (1000)(9.8)(0.5 - 8.03\times 10^{-8} y)\ N$$
   $$= (4900 - 7.87 y)\ N$$

   (c) Is there ever a depth at which the Buoyancy force equals the gravitational pull on the brass sphere, in which case it will begin to stop sinking? If so, find that depth.

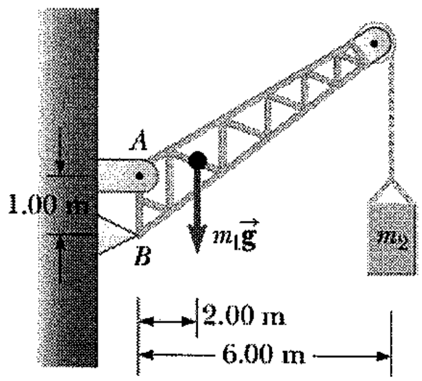
   $$\rho_{Brass} = 8.4\times 10^3\ \frac{kg}{m^3} \rightarrow M_B = \rho V = (8.4\times 10^3)(0.5) = 4200\ kg$$
   $$W_B = M_B g = 41{,}160\ N$$

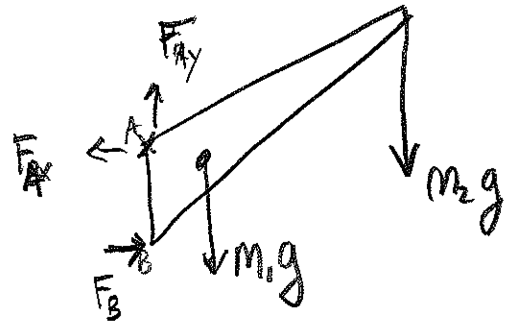
   Never will $\mathbb{B} > W_B$

   (d) Think back to problem 7. Is our assumption that the density of water doesn't change reasonable for this problem? If you found a depth in part (c), is that reasonable? The deepest point in the ocean is about 11 km.

   It will never happen. Assuming Water density is the same is reasonable.

10. 12 points A crane of mass $m_1 = 3000$ kg supports a load of mass $m_2 = 10000$ kg. The crane is pivoted with a frictionless pin at A (rotates at A) and rests against a smooth (that means frictionless) support at B. Find the reaction forces of points A and B. *Hint: Here, using the* **action radius** *which is more commonly called the* **lever arm** *will be the quickest route.*





$$\Sigma F_x = F_B - F_{Ax} = 0$$

$$\Sigma F_y = F_{Ay} - M_1 g - M_2 g = 0$$

$$\Sigma \tau = -6(m_2 g) - 2(m_1 g) + 1(F_B) = 0$$

$$-6(98000) - 2(29400) + F_B = 0$$

$$\boxed{F_B = 646800\text{ N}} \uparrow$$

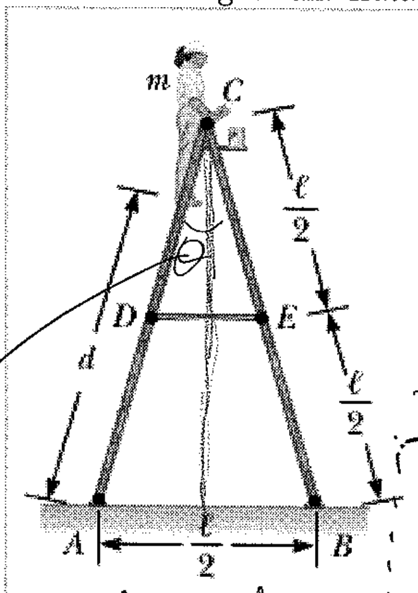
$$F_B - F_{Ax} = 0 \rightarrow$$

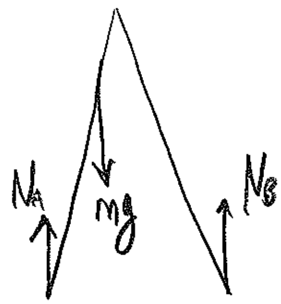
$$F_{Ax} = F_B = \underline{646800\text{ N}}$$

$$F_{Ay} - 29400 - 98000 = 0$$

$$\underline{F_{Ay} = 127400\text{ N}}$$

11. $\boxed{\text{12 points}}$ Solve this problem without numbers, just variables. It will be useful to acclimate yourself to this request as your career progresses. Consider the ladder in the figure below. Find (a) the tension in the horizontal bar DE, (b) the normal forces at A and B, and all the reaction forces at hinge C that the left half of the ladder exerts on the right half. *Hint: Treat the ladder both as two halves and as a single unit.*



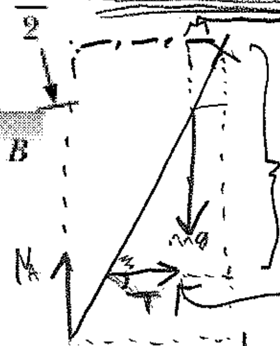


$\Sigma F_x = 0$

$\Sigma F_y = 0 = N_A + N_B - mg$

$N_A + N_B = mg$

$|-\ell/4-|-\ell/4-|$



$\tau_3 = mg(\ell - d)(\sin(\theta))$

$= mg\frac{(\ell-d)}{4}$

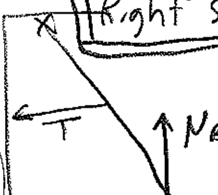
$\tau_2 = T\frac{\ell}{2}\cos(\theta) = T\frac{\sqrt{15}}{8}\ell$

$\tau_1 = N_A \frac{\ell}{4}$

$\sin(\theta) = \frac{\frac{\ell}{4}}{\ell} = \frac{1}{4}$

$\cos(\theta) = \frac{\frac{\sqrt{15}}{4}\ell}{\ell} = \frac{\sqrt{15}}{4}$

$\boxed{\text{Right side}}$



$\Sigma \tau = -T\frac{\sqrt{15}}{8}\ell + N_B\frac{\ell}{4} = 0$

$\boxed{T = N_B\frac{2}{\sqrt{15}}}$

$\boxed{\text{Left}}$ $\Sigma \tau = -N_A\frac{\ell}{4} + T\frac{\sqrt{15}}{8}\ell + mg\frac{\ell-d}{4} = 0$

$= -N_A\ell/4 + N_B\frac{\ell}{4} + (N_A + N_B)\frac{\ell-d}{4} = 0$

$-N_A d + (2\ell - d)N_B = 0$

$N_A = \frac{2\ell - d}{d}N_B$ $\xrightarrow{}$ $\boxed{N_A + N_B = mg}$

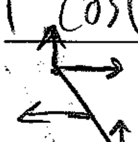
$\left(\frac{2\ell-d}{d} + 1\right)N_B = mg$

$\sqrt{\ell^2 - \ell^2/16} = \frac{\sqrt{15}}{4}\ell$

$\ell/4$

$\sin(\theta) = \frac{\text{opp.}}{\text{hypot.}} = \frac{\ell/4}{\ell} = 1/4$

$\cos(\theta) = \text{adj}/\text{hypot} = \sqrt{15}/4\ \ell/\ell = \sqrt{15}/4$



$F_{cx} = T$

$F_{cy} = N_B$

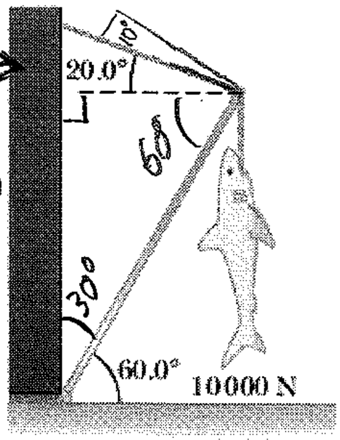
For C, easiest to look at right side.

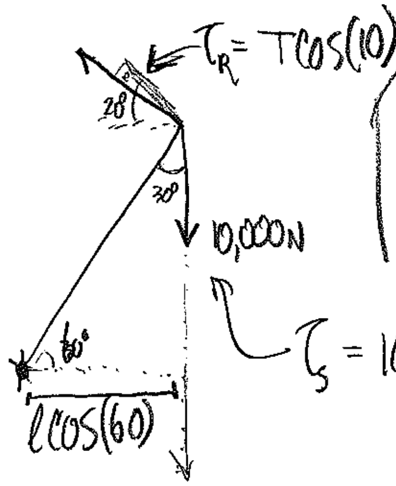
$\boxed{T = \frac{d\,mg}{\sqrt{15}\,\ell}}$ $\leftarrow$ $\boxed{N_B = \frac{d\,mg}{2\ell}}$ $\rightarrow$ $\boxed{N_A = \frac{2\ell - d}{2\ell}mg}$

Super Long Problem

12. $\boxed{\text{20 points}}$ Consider the following scenario: A 10,000-N shark is supported by a rope attached to a 4.00-m rod (negligible mass) that can pivot at the base.

$T_x = T\cos(20)$
$T_y = T\sin(20)$

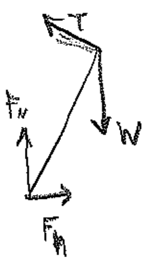




lever-arm is easier for shark, not for tension.

$\tau_s = 10{,}000N(l\cos(60))$

(a) Calculate the tension in the cable between the rod and the wall, assuming the cable is holding the system in the position shown.



$$\Sigma F_x = F_h - T\cos(20) = 0$$
$$\Sigma F_y = T\sin(20) + F_v - 10000\,N = 0$$
$$\Sigma \tau = 4(T\cos(10)) - 10{,}000[4\cos(60)] = 0 \rightarrow T = 5080\,N$$

(b) Find the horizontal force exerted on the base of the rod.

$$F_h - T\cos(20) = 0 \rightarrow F_h = T\cos(20)$$
$$= 4800\ N$$

(c) Find the vertical force exerted on the base of the rod.

$$F_v = -T\sin 20 + 10000 = 8260\,N$$

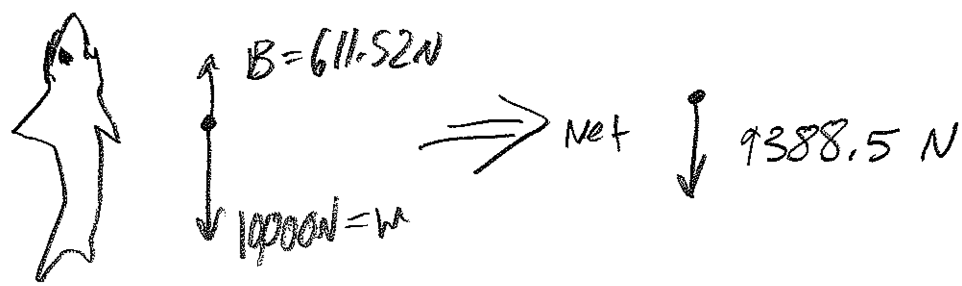
(d) Assume that the cable is made out of Aluminum with a Young's Modulus of 7.0×10^{10} N/m^2. By how much will it stretch?

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} \rightarrow \Delta L = \frac{FL}{AY} = \frac{(5080\,N)L}{A(7.0\times10^{10}\,N/m^2)} = 7.3\times10^{-8}\frac{L}{A}$$

Depends on length and inversely on cross-sectional area

(e) Assume that the shark has a volume of 0.0624 m^3. Suppose the tank is filled with water so that the shark is submerged. Recalculate everything in parts a, b, c, and d given these new conditions (hint: Buoyancy). What is the benefit of submerging the shark in water?

$$B = \rho_w g V_{displaced}$$
$$= 1000\,kg/m^3 (9.8\,m/s^2)(0.0624\,m^3)$$
$$= 611.52\,N$$



Replace 10000N with 9388.5N in all previous equations.

a) 4766 N

b) 4479 N

c) 7758 N

d) $6.8 \times 10^{-8}\ L/A$

$$\left.\frac{7.3\times10^{-8} - 6.8\times10^{-8}\ L/A}{7.3\times10^{-8}\ L/A} = 0.068\right] \sim 7\%\ \text{less stretching}$$

End of exam