Physics 185F2013 Lecture Three

October 8, 2013

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Consequences of Newton's Laws

A refresher:

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- Velocity is not the same as speed. Velocity is speed plus direction. A car going 20 mph is not necessarily the same as a car going 20 mph North.
- If two objects with different masses are going the same speed, the one with the higher mass has more momentum.

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- Average speed: $s = \frac{\Delta x}{\Delta t}$, the average speed over an entire trip; Instantaneous speed: $\frac{dx}{dt}$, how fast something is going at any given time.
- To make speed a velocity, add a direction. $v = 20 m/s$ North.

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- Second Law: F = ma
- Third Law: To every action there is always opposed an equal reaction

Now for the new stuff:

This week we focus on the second law, $F = ma$. What are the consequences of this law?

- Think Calculus

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- Let x_1 be the position of the object at time t_1.
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- The object's positioned has changed $\Delta x = x_2 - x_1$ (Displacement) in time $\Delta t = t_2 - t_1$.
- For example, if an object started at the 2m mark, we start a stopwatch at $t_1 = 0$ and then stop it when the watch reads $t_2 = 5s$ at which point the object is at $x_2 = 5m$, then $\Delta x = 5m - 2m = 3m$ and $\Delta t = 5s - 0s = 5s$.

Average Speed

Average speed is defined in terms of the total distance traveled by an object, and not Δx:

$$\bar{\text{speed}} = \frac{s}{\Delta t} \neq \frac{\Delta x}{\Delta t}$$

Average Velocity

Average velocity is defined as the total displacement divided by the change in time:

$$\bar{\mathrm{v}} = \frac{\Delta \mathrm{x}}{\Delta \mathrm{t}}$$

Instantaneous velocity

To define instantaneous velocity, we need calculus. If x is the position of the object as a function of time, then

$$\mathrm{v} = \lim_{\Delta \mathrm{t} \to 0} \frac{\Delta \mathrm{x}}{\Delta \mathrm{t}} = \frac{\mathrm{dx}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{x})$$

- If the particles position at any point in time is described by $\mathrm{x} = 2\mathrm{t}^2$, how fast is the particle going at $\mathrm{t} = 3\mathrm{s}$?

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- At 3s, $v_3 = 4 \times 3 = 12$ m.

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$$F_g = mg = (2\ \mathrm{kg})\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 19.6\frac{\mathrm{kg\ m}}{\mathrm{s}^2} = 19.6\ \mathrm{N}$$

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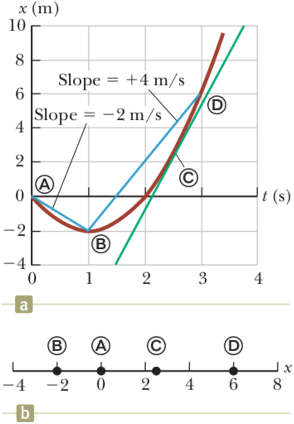
But we also know that $\mathrm{F} = \mathrm{ma}$ so that

$$(2\ \mathrm{kg})\,\mathrm{a} = \mathrm{F} = 19.6\ \mathrm{N} \ \rightarrow\ \mathrm{a} = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$$

This tells us that the block will accelerate downwards towards the Earth at that rate. Put that thought in the back of your mind for a moment.

Example 2



Q.

A particle moves along the x-axis with position given as a function of time as $x = -4t + 2t^2$. Find the displacement of the particle between $t = 0$ and $t = 1s$.

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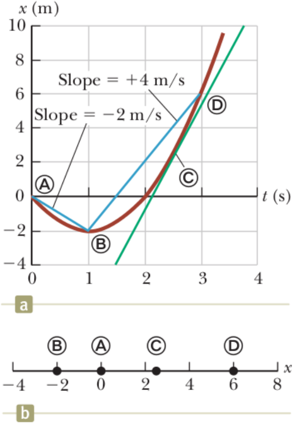

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\begin{aligned}
x_1 &= \left(-4 \times 0 + 2 \times 0^2\right) \mathrm{m} \\
x_2 &= \left(-4 \times 1 + 2^2\right) = -2\mathrm{m} \\
\Delta x &= x_2 - x_1 = -2 - 0 = -2\mathrm{m}
\end{aligned}
$$

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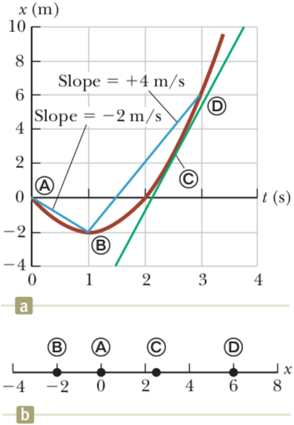

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A particle moves along the x-axis with position given as a function of time as $x = -4t + 2t^2$. Find the displacement of the particle between t = 1 and t = 3s.

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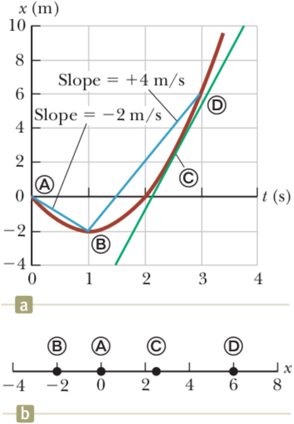

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A.

$$\begin{aligned} \mathrm{x}_1 &= \left(-4 \times 1 + 2 \times 1^2\right) = -2\mathrm{m} \\ \mathrm{x}_3 &= \left(-4 \times 3 + 2^2\right) = 6\mathrm{m} \\ \Delta\mathrm{x} &= \mathrm{x}_3 - \mathrm{x}_1 = 6 - (-2) = 8\mathrm{m} \end{aligned}$$

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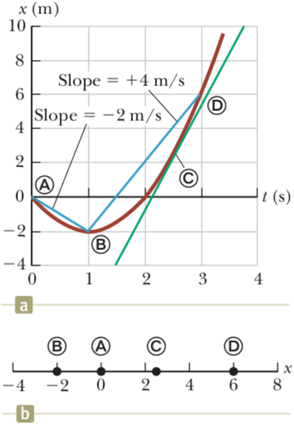

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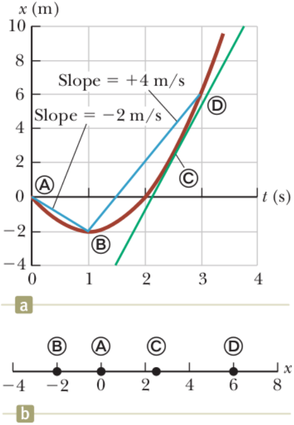

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$$\bar{v}_{0,1} = \frac{x_1 - x_0}{t_1 - t_0} = \frac{-2\ \mathrm{m}}{1\ \mathrm{s}} = -2\frac{\mathrm{m}}{\mathrm{s}}$$

$$\bar{v}_{1,3} = \frac{x_3 - x_1}{t_3 - t_1} = \frac{8\mathrm{m}}{2\ \mathrm{s}} = 4\frac{\mathrm{m}}{\mathrm{s}}$$

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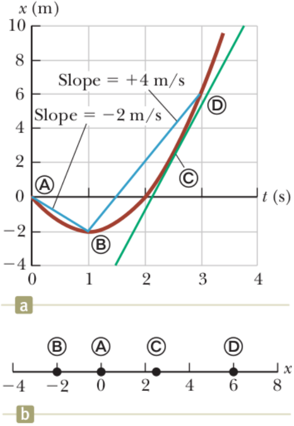

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Find the instantaneous velocity of the block at t = 0, t = 1, and t = 3 seconds. Recall that the objects position is given as a function of time as $x = -4t + 2t^2$ so that $v = \frac{d}{dt}\left(-4t + 2t^2\right) = -4 + 4t$.

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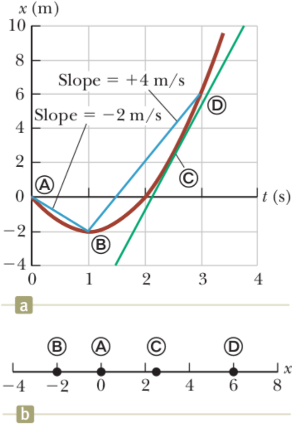

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$$\begin{aligned} v_0 &= -4 + 4 \times 0 = -4\frac{m}{s} \\ v_1 &= -4 + 4 \times 1 = 0\frac{m}{s} \\ v_3 &= -4 + 4 \times 3 = 8\frac{m}{s} \end{aligned}$$

Average Acceleration

Average acceleration is defined as the change in velocity divided by the change in time:

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Suppose we release it from rest 2 meters above the ground. Can we predict when it will hit the ground?

Integration

Recall that

$$\frac{d}{dt}t^2 = 2t$$

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$$\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{t}^2 = 2\mathrm{t}$$

and

$$\int 2\mathrm{t}\,\mathrm{dt} = \mathrm{t}^2 + \mathrm{c}$$

where c is some unknown constant. This is a Calculus-based course, but our use of Calculus isn't going to get much more harder than that.

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What is c? Since we are releasing the block, that means that its original velocity was zero. Then at $\mathrm{t} = 0$ we need $\mathrm{v} = 0$, so we set $\mathrm{c} = 0$ and we have:

$$\mathrm{v} = -9.8\mathrm{t}$$

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So we figure out that the 2.0 kg block experiences an acceleration of $9.8\frac{\mathrm{m}}{\mathrm{s}^2}$ downward when dropped from a height of 2.0 meters. We just figured out that the velocity will be $\mathrm{v} = -9.8\mathrm{t}$. What about its position?

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$$v = \frac{dx}{dt} \;\rightarrow\; x = \int v\,dt$$

What is the position as a function of time for the block?

$$x = \int v\,dt = \int -9.8t\,dt = \frac{1}{2}\left(-9.8t^2\right)\mathrm{m} + c$$

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What is c? Since the original position of the block is 2.0 meters at time $t = 0$, we require:

$$x_0 = \frac{-9.8}{2}0^2 + c = 0 + c = 2m$$

Then our x function is:

$$x = \frac{-9.8}{2}t^2 + 2$$

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$$0 = \frac{-9.8}{2}\mathrm{t}^2 + 2$$

$$\sqrt{\frac{-4}{-9.8}} = \mathrm{t} \quad \rightarrow \quad \mathrm{t} = 0.64 \text{ s}$$

Motion in 1-d, in general, constant acceleration

In general, given a constant acceleration a that doesn't depend on time, the velocity will be:

$$\mathrm{v} = \int \mathrm{a}\,\mathrm{dt} = \mathrm{at} + \mathrm{v}_0$$

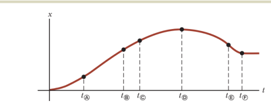
where v_0 is the starting velocity, usually at time $\mathrm{t} = 0$, and in general, pause

$$\mathrm{x} = \int \mathrm{v}\,\mathrm{dt} = \int (\mathrm{at} + \mathrm{v}_0) = \frac{1}{2}\mathrm{at}^2 + \mathrm{v}_0\mathrm{t} + \mathrm{x}_0$$

where x_0 is the starting position, usually at time $\mathrm{t} = 0$.

Acceleration, Velocity, and Position graphs

The graph shows the x position of an object along the x-axis. Plot a corresponding velocity and acceleration graph.



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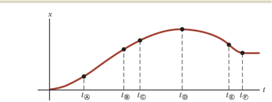
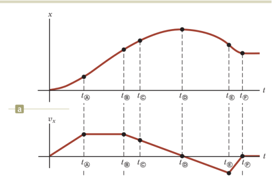
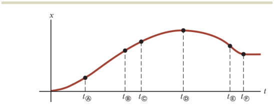
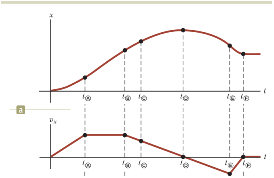


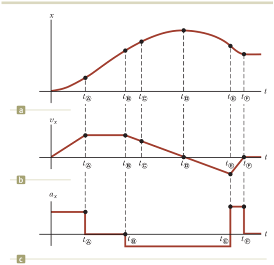


Acceleration, Velocity, and Position graphs

The graph shows the x position of an object along the x-axis. Plot a corresponding velocity and acceleration graph.







Collecting the equations

The Kinematic Equations of Motion for constant acceleration in one dimension are:

$$
\begin{aligned}
\mathrm{v_f} &= \mathrm{v_i} + \mathrm{at} && (1)\\
\mathrm{x_f} &= \mathrm{x_i} + \mathrm{v_i t} + \frac{1}{2}\mathrm{at}^2 && (2)\\
\mathrm{x_f} &= \mathrm{x_i} + \frac{1}{2}\left(\mathrm{v_i} + \mathrm{v_f}\right)\mathrm{t} && (3)\\
\mathrm{v_f^2} &= \mathrm{v_i^2} + 2\mathrm{a}(\mathrm{x_f} - \mathrm{x_i}) && (4)
\end{aligned}
$$

Kinematic Equation examples

Classroom examples on board in groups.

Recitation Problems: Chapter 1: 53

(Draw diagrams from board on this paper if needed).

- 53.a) the +y axis is 90 degrees from zero, so this is a 120 degree angle: $x = 10\cos(120) = -5.0$ m and $y = 10\sin(120) = 8.7$ m.

Recitation Problems: Chapter 1: 53

(Draw diagrams from board on this paper if needed).

- 53.a) the +y axis is 90 degrees from zero, so this is a 120 degree angle: $x = 10\cos(120) = -5.0$ m and $y = 10\sin(120) = 8.7$ m.
- 53.b) 40 degrees Counterclockwise from -x = 180 + 40 = 220. $x = 25\cos(220) = -19$ m/s, $y = 25\sin(220) = -16$ m/s

Recitation Problems: Chapter 1: 53

(Draw diagrams from board on this paper if needed).

- 53.a) the +y axis is 90 degrees from zero, so this is a 120 degree angle: $x = 10\cos(120) = -5.0$ m and $y = 10\sin(120) = 8.7$ m.
- 53.b) 40 degrees Counterclockwise from -x = 180 + 40 = 220. $x = 25\cos(220) = -19$ m/s, $y = 25\sin(220) = -16$ m/s
- 53.c) 120 counterclockwise from -y: -y is at 270, so 270+120=390=360+30. Thus this is at 30 degrees. $x = 40\cos(30) = 35$ lb, $y = 40\sin(30) = 20$ lb.

Recitation Problems: Chapter 1: 57

(Draw diagrams from board on this paper if needed).

$$\vec{\mathrm{A}} = 3.4\hat{\mathrm{i}} + 4.7\hat{\mathrm{j}}, \quad \vec{\mathrm{b}} = -7.7\hat{\mathrm{i}} + 3.2\hat{\mathrm{j}}, \quad \vec{\mathrm{C}} = 5.4\hat{\mathrm{i}} - 9.1\hat{\mathrm{j}}$$

Recitation Problems: Chapter 1: 57

(Draw diagrams from board on this paper if needed).

$$\vec{\mathrm{A}} = 3.4\hat{\mathrm{i}} + 4.7\hat{\mathrm{j}}, \quad \vec{\mathrm{b}} = -7.7\hat{\mathrm{i}} + 3.2\hat{\mathrm{j}}, \quad \vec{\mathrm{C}} = 5.4\hat{\mathrm{i}} - 9.1\hat{\mathrm{j}}$$

Recitation Problems: Chapter 1: 57

(Draw diagrams from board on this paper if needed).

$$\vec{\mathrm{A}} = 3.4\hat{\mathrm{i}} + 4.7\hat{\mathrm{j}}, \quad \vec{\mathrm{b}} = -7.7\hat{\mathrm{i}} + 3.2\hat{\mathrm{j}}, \quad \vec{\mathrm{C}} = 5.4\hat{\mathrm{i}} - 9.1\hat{\mathrm{j}}$$

$$\vec{\mathrm{D}} + 2\vec{\mathrm{A}} - 3\vec{\mathrm{C}} + 4\vec{\mathrm{B}} = 0 \quad \Longrightarrow \quad \vec{\mathrm{D}} = 2\vec{\mathrm{A}} - 3\vec{\mathrm{C}} + 4\vec{\mathrm{B}}$$

Recitation Problems: Chapter 1: 57

(Draw diagrams from board on this paper if needed).

$$\vec{\mathrm{A}} = 3.4\hat{\mathrm{i}} + 4.7\hat{\mathrm{j}}, \quad \vec{\mathrm{b}} = -7.7\hat{\mathrm{i}} + 3.2\hat{\mathrm{j}}, \quad \vec{\mathrm{C}} = 5.4\hat{\mathrm{i}} - 9.1\hat{\mathrm{j}}$$

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$$\vec{\mathrm{D}} = [2(3.4) - 3(5.4) - 4(7.7)]\,\hat{\mathrm{i}} + [2(4.7) + 3(9.1) + 4(3.2)]\,\hat{\mathrm{j}} = -40\hat{\mathrm{i}} + 50\hat{\mathrm{j}}$$

Recitation Problems: Chapter 4: 47

(Draw diagrams from board on this paper if needed). Form two equations:

$$\mathrm{T}_2 \sin(60) + \mathrm{T}_1 \sin(30) = (35\ \mathrm{kg})\, 9.8 \frac{\mathrm{m}}{\mathrm{s}^2}$$

Recitation Problems: Chapter 4: 47

(Draw diagrams from board on this paper if needed). Form two equations:

$$\mathrm{T}_2 \sin(60) + \mathrm{T}_1 \sin(30) = (35\ \mathrm{kg})\, 9.8 \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\mathrm{T}_2 \cos(60) = \mathrm{T}_1 \cos(30)$$

Recitation Problems: Chapter 4: 47

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Solve for T_1 in terms of T_2:

Recitation Problems: Chapter 4: 47

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Solve for T_1 in terms of T_2:

$$\mathrm{T}_1 = \frac{\cos(60)}{\cos(30)} \mathrm{T}_2$$

Recitation Problems: Chapter 4: 47

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$$\mathrm{T}_2 \sin(60) + \mathrm{T}_1 \sin(30) = (35\ \mathrm{kg})\, 9.8 \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\mathrm{T}_2 \cos(60) = \mathrm{T}_1 \cos(30)$$

Solve for T_1 in terms of T_2:

$$\mathrm{T}_1 = \frac{\cos(60)}{\cos(30)} \mathrm{T}_2$$

Plug into the first equation, factor out T_2 to find $\mathrm{T}_2 = 289\mathrm{N}$ and $\mathrm{T}_1 = 172\mathrm{N}$. $\mathrm{T}_2 > \mathrm{T}_1$.

Recitation Problems: Chapter 4: 51

(Draw diagrams from board on this paper if needed). Add up the x-components and y-components. The third force must be the negative of that. The only one that takes extra work is F_2, which we decompose into $F_{2,x} = 30\text{N}\cos(30)$ and $F_{2,y} = 30\text{N}\sin(30)$ so that the new force must be $-26\text{N}\hat{\imath} - 5\text{N}\hat{\jmath}$.

Recitation Problems: Chapter 4: 51

In the y-direction, $2\mathrm{T}\sin(3) = 400\mathrm{N} \implies \mathrm{T} = 3.82\mathrm{kN}$ for part (a), and $2\mathrm{T}\sin(4) = 600\mathrm{N} \implies \mathrm{T} = 4.3\mathrm{kN}$ for part (b).