Physics 185F2013 Lecture Six

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What is force, again?

- Force is vector quantity that measures the amount of influence something has on the state of motion (speed and/or direction) or physical state of an object (bending).
- Newton's second law says:
- $\sum F_x = ma_x$ and $\sum F_y = ma_y$.
- If the forces all balance, then $a_x = 0$ and $a_y =$ and the object is said to be in equilibrium.

What is work?

Recall that just having a force on an object doesn't mean that something will happen to that object.

- We need a more flexible concept that captures the ability of force to do something that will change the object's state in such a way that the object can change other objects.
- For example, a block sitting on a table is doing nothing. If you push that block, then it can slide and hit another block, causing that block to move. You have done work on the block. The block now has more energy–which is the potential to do work, and eventually does work on the second block.

Work is defined as (in the x-direction):

$$\mathrm{W} = \int_{\mathrm{x}_1}^{\mathrm{x}_2} \mathrm{F}_\mathrm{x} \mathrm{d}\mathrm{x}$$

We can generalize for other dimensions. Whenever the force is constant, that is, it doesn't depend on the position x, we have more simply:

$$\mathrm{W} = \mathrm{F}\Delta\mathrm{x}$$

An example

You lift a 1 kg book 2 meters off of a table. How much work did you do? How much work did gravity do? Assume constant force and that you only apply enough force to compensate for gravity.

Your work: [Draw free body here], you needed to overcome the books weight of 9.8 N, and you did so over 2 meters, so you did work of

$$\mathrm{W_{you} = F\Delta y = 9.8N(2m) = 19.6Nm}$$

Earth's work: The Earth was pulling down in the opposite direction of the displacement, so we write its force as negative:

$$\mathrm{W_{E} = F\Delta y = -9.8N(2m) = -19.6Nm}$$

Total work done on the book is zero. The units Nm are called Joules (J) so that, for example, gravity did -19.6 J on the book.

What is power?

Power is the rate at which work is done.

From the pervious example, if you had done 19.6 Nm of work in 10 seconds, you would have exerted a power of

$$\mathrm{P} = \frac{\Delta \mathrm{W}}{\Delta \mathrm{t}} = \frac{19.6\mathrm{Nm}}{10\mathrm{s}} = 1.96\mathrm{W}$$

The unit for power is called a Watt (W) and is made of J/s.

If you had lifted it at a constant rate of 5 m/s, since you lifted it 2 meters, it would have taken 2/5 seconds for a work rate of $\frac{19.6}{2/5} = \frac{19.6(5)}{2} = 9.8(5)$. In other words, we alos have,

$$\mathrm{P} = \mathrm{Fv}$$

When v is constant.

Power in general

Recall that,

$$W = \int_{x_1}^{x_2} F dx$$

In general, we can write power as,

$$P = \frac{dW}{dt} = \frac{d}{dt}(F dx) = F \frac{dx}{dt} = Fv$$

What if the net work is not zero?

Now suppose that you had lifted the book with 15 N instead of 9.8 N?

By Newton's second law, this would result in a net acceleration upwards. That is, the speed of the book would change. Find the speed of the book using the laws of kinematics once it has been lifted 2 meters.

What if the net work is not zero?

Now suppose that you had lifted the book with 15 N instead of 9.8 N?

In terms of work and energy, we say that we have put work into the system, and this will increase its energy–specifically, its Kinetic Energy.

$$\mathrm{W} = \int \mathrm{F dx} = \int \frac{\mathrm{dP}}{\mathrm{dt}} \mathrm{dx} = \int \mathrm{dP} \frac{\mathrm{dx}}{\mathrm{dt}}$$

$$= \int \mathrm{d(mv)v} = \mathrm{m} \int \mathrm{vdv} = \frac{\mathrm{m}}{2} \left(\mathrm{v_f^2} - \mathrm{v_i^2} \right)$$

If we define the kinetic energy of a particle to be $\mathrm{K} = \frac{\mathrm{m}}{2}\mathrm{v}^2$, then we have the Work-Kinetic-Energy Theorem: $\mathrm{W_{Total}} = \Delta \mathrm{K}$

One more thing about that book...

There's something missing from our energy discussion. If you lift that book 2 meters off the table, what happens when you drop it? It falls back to the table. Thus it has apparently been given some form of energy. But isn't the net work done equal to zero?

Gravity is a special type of force. It is not a contact force, it is a field force. Also, it is a conservative force: the work done by gravity is independent of the path taken as the book moves from point A to point B, and in fact, the net work done is zero if the book returns to point A.

One more thing about that book...

The book really "wants" to get back to point A if possible. As soon as you release it, it will fall back to where it started. As it falls, it gets faster and thus increases kinetic energy. Where did that energy come from? Gravity.

When the book is held two meters off the table, it has been given "potential energy", which is converted into kinetic energy as it falls back to the table.

$$\mathrm{Potential\ Energy} = \Delta \mathrm{U} = -\int_{\mathrm{x}_1}^{\mathrm{x}_2} \mathrm{F}\mathrm{d}\mathrm{x} = \mathrm{mg}\Delta \mathrm{y}$$

One more thing about that book...

Gravity did -19.6 J of work on the book as we lifted it up, thus investing it with 19.6 J of Potential Energy. We can be just as sure that when it falls back to the table, the potential energy will have been converted into 19.6 J of Kinetic Energy.

Total work

The total work on a system is the work done by external forces, such as you lifting a book, plus work done by non-conservative forces that are part of the system, plus work done by conservative forces that are part of the system such as gravity. For now, we ignore the non-conservative forces that part of the system. Combining everything, we have the work-energy theorem:

$$\mathrm{W_{ext}} = \Delta\mathrm{K} + \Delta\mathrm{U} = \Delta\mathrm{E_{mech}}$$

where

$$\Delta\mathrm{E_{mech}} = \Delta\mathrm{K} + \Delta\mathrm{U}$$

is the change in “mechanical energy”, or the sum of kinetic and potential energy changes.

One more thing about that book...

In our book problem, we don't have to determine the work done by gravity, we can instead determine the change in potential energy:

$$\mathrm{W_{ext}} = \Delta\mathrm{K} + \Delta\mathrm{U} = 0 + \mathrm{mgh} = 9.8\mathrm{N}(2\mathrm{m}) = 19.6\mathrm{J}$$

Conservation of Energy

If no external forces act on a system, thus there is no external work done, then the mechanical energy of the system will not change:

$$\mathrm{W}_{\mathrm{ext}} = 0 \implies 0 = \Delta\mathrm{E}_{\mathrm{mech}}$$

Elastic Collisions

- Last week we talked about inelastic collisions and conservation of linear momentum.
- Now we can talk about elastic collisions which require conservation of Kinetic Energy.
- $\mathrm{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$
- $\frac{1}{2}\mathrm{m_1 v_{1i}^2} + \frac{1}{2}\mathrm{m_2 v_{2i}^2} = \frac{1}{2}\mathrm{m_1 v_{1f}^2} + \frac{1}{2}\mathrm{m_2 v_{2f}^2}$
- We can factor out the 1/2 and shuffle:
- $\mathrm{m_2(v_{2f}^2 - v_{2i}^2) = m_1(v_{1i}^2 - v_{1f}^2)}$
- $\mathrm{(a^2 - b^2) = (a - b)(a + b)}$ so we write:
- $\mathrm{m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) = m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f})}$

Elastic Collisions

- But also:
- $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
- $m_2(v_{2f} - v_{2i}) = m_1(v_{1i} - v_{1f})$,
- we divide the above result and $m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) = m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f})$ to find:
- $v_{1i} - v_{2i} = v_{2f} - v_{1f}$
- or:
- $v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$
- or:
- $v_{2i} + v_{2f} = v_{1i} + v_{1f}$

Elastic Collisions

After some algebra, we have the following equations for elastic collisions (note that your book doesn't present these equations in full):

$$\mathrm{v_{1f}} = \frac{\mathrm{m_1 - m_2}}{\mathrm{m_1 + m_2}}\mathrm{v_{1i}} + \frac{\mathrm{2m_2}}{\mathrm{m_1 + m_2}}\mathrm{v_{2i}}$$

and

$$\mathrm{v_{2f}} = \frac{\mathrm{2m_1}}{\mathrm{m_1 + m_2}}\mathrm{v_{1i}} + \frac{\mathrm{m_2 - m_1}}{\mathrm{m_1 + m_2}}\mathrm{v_{2i}}$$