Physics 185F2013 Lecture Three

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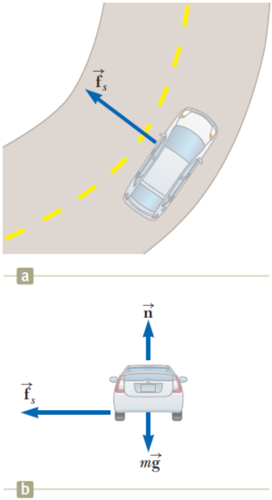
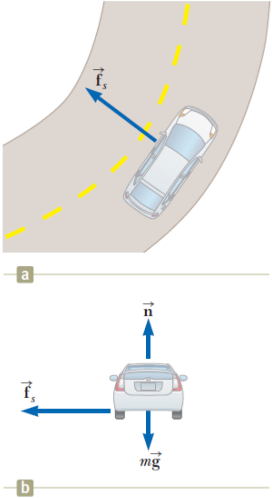
Inertial Frames



- Imagine you place your cell phone on the dashboard of your car. You take a sharp turn around a curve. What happens to the cell phone? Why?
- In our previous work, we assumed that we were working in a Non-inertial reference frame.
- = a frame which is not accelerating
- Earth is not a non-inertial reference frame.

Inertial Frames



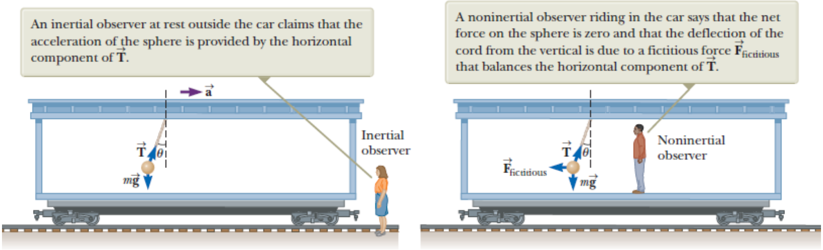
Inertial Frame

A frame that is not accelerating. Newton's laws will hold precisely in such a frame. The Earth is accelerating as it travels around the sun and as it rotates and wobbles, so it isn't an inertial frame.

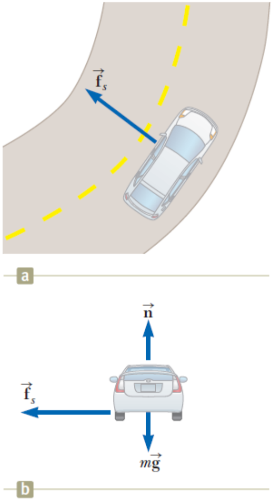
Fictitious Force

A fictitious force is a force which is seen in a non-inertial frame which doesn't seem to hold to Newton's Laws. Normal forces happen between two objects, fictious force appears to act on an object alone– no second object can be identified.

Two observers, two different stories



Inertial Frames



Example

As you make a turn in a car, objects in the car will slide as if acted upon by a force, though there is no source.

Centripetal acceleration

$$a = \frac{v^2}{R} \tag{1}$$

Always along the radius connecting a body to its center of rotation.

Example 1

Engineers design on-ramps and off-ramps of major highways to have a significant slant or "bank". Why would they do this? Consider a ramp with speed-limit 30.0 mi/h (13.4 m/s) and a radius of curvature of 35.0m. If you are designing this road, what angle would you bank this turn with? Draw free body diagrams.

Example 2

Design a Ferris Wheel that can handle up to 1000 kg per car. Draw free body diagrams.

Example 3

Suppose you are navigating a rocket in deep space and you want to go in a circle of radius 3,053 meters at a speed of 100 m/s. By how much and in what direction should you accelerate the rocket?

Coriolis Force

We don't have the time to go into detail on this fiction force, but you will get a bonus point on the test if you can write a few sentences to describe it, so do a web search on basic facts of the Coriolis force.

Linear and Angular physics

	Translational Motion		Rotational Motion	
	Symbol	**Units**	**Symbol**	**Units**
Position	x	(m)	θ	(rad)
Speed	$v = \frac{dx}{dt}$	(m/s)	$\omega = \frac{d\theta}{dt}$	(rad/s)
Acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	(m/s^2)	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	(rad/s^2)
Momentum	$p = mv$	(kgm/s)	$L = I\omega$	(kgm^2/s)

Source of	**Parameter**	**Symbol**	**Units**	**Parameter**	**Symbol**	**Units**
Acceleration	Force	$F = ma = \frac{dp}{dt}$	(N)	Torque	$\tau = I\alpha = \frac{dL}{dt}$	(Nm)
Inertia	Mass	m	(kg)	Moment of Inertia	$I = k\left(mr^2\right)$	(kgm^2)

Linear and Angular physics

Rigid Body Under Constant Angular Acceleration	Particle Under Constant Acceleration
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Linear and Angular physics

Everything we've learned so far translates almost easily into angular form. The hardest part is finding the equivalence of inertia. Let's relate translational (or linear) kinetic energy to angular kinetic energy for a particle going in a circle with radius R:

$$\mathrm{K} = \frac{1}{2}\mathrm{mv}^2 = \frac{1}{2}\mathrm{m}(\mathrm{R}\omega)^2 = \frac{1}{2}\mathrm{mR}^2\omega^2 = \frac{1}{2}\left(\mathrm{mR}^2\right)\omega^2$$

This tells us that for a single rotating particle, the equivalent inertia is

$$\mathrm{I} = \mathrm{mR}^2$$

Moment of Inertia

In general (often there is more than one single particle), we will define the angular equivlence of inertia (called the “Moment of Inertia”):

$$I = \sum_i m_i r_i^2$$

For continuously distributed mass, such as in a solid body, we have:

$$I = \int r^2 dm$$

where the integral is over the entire body.

Moment of Inertia

Find the moment of inertia for a thin rod about perpendicular line through center of rod?
The rod has a total mass of M distributed evenly over a length L, so the mass per unit length is M/L, and so

$$\mathrm{dm} = \frac{\mathrm{M}}{\mathrm{L}}\mathrm{dr}$$

$$\mathrm{I} = \int_{-\mathrm{L}/2}^{\mathrm{L}/2} \mathrm{r}^2 \mathrm{dm} = \frac{\mathrm{M}}{\mathrm{L}} \int_{-\mathrm{L}/2}^{\mathrm{L}/2} \mathrm{r}^2 \mathrm{dr} = \frac{\mathrm{M}}{3\mathrm{L}} \left[\frac{\mathrm{L}^3}{8} + \frac{\mathrm{L}^3}{8} \right] = \frac{1}{12}\mathrm{ML}^2$$

Parallel-Axis Theorem

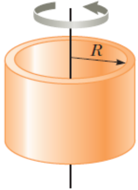
According to this theorem, we can find the moment of inertia of a body around any axis by relating it to a parallel axis that passes through the center of that body.

$$\mathrm{I} = \mathrm{I}_{\mathrm{cm}} + \mathrm{Mh}^2$$

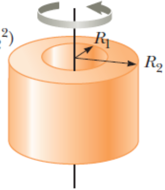
Use this theorem to calculate the moment of inertia of a thin rod about an axis taken from one of its ends.

Moments of Inertia

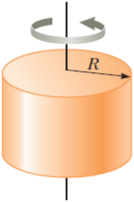
Hoop or thin cylindrical shell
$I_{CM} = MR^2$



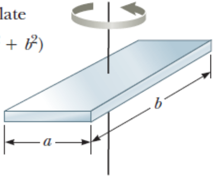
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^{\,2} + R_2^{\,2})$



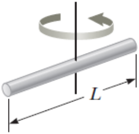
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$



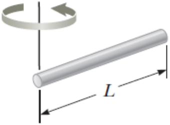
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



Long, thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$



Long, thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$

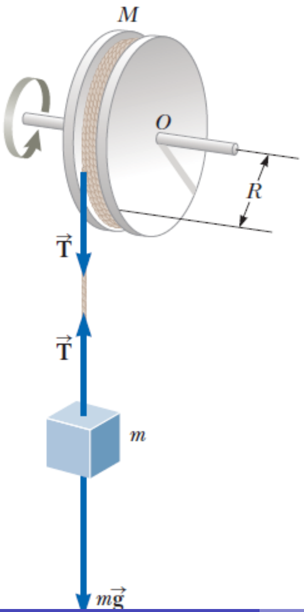


Newton's Second Law (Rotation)

Now that we've achieved a definition for inertia in terms of rotation, we can use Newton's Laws for scenarios involving rotation.
Torque is defined to be $\tau = \vec{\mathrm{F}}\mathrm{R}$, which is the rotational equivalent of $\mathrm{F} = \mathrm{ma}$. Newton's second law in rotation form is:

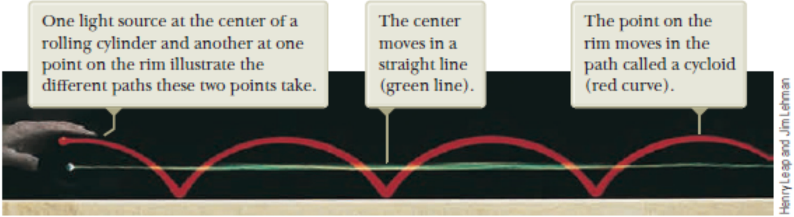
$$\sum \tau_{\mathrm{ext}} = \mathrm{I}\alpha$$

Example 4: Non-Ideal Pulley

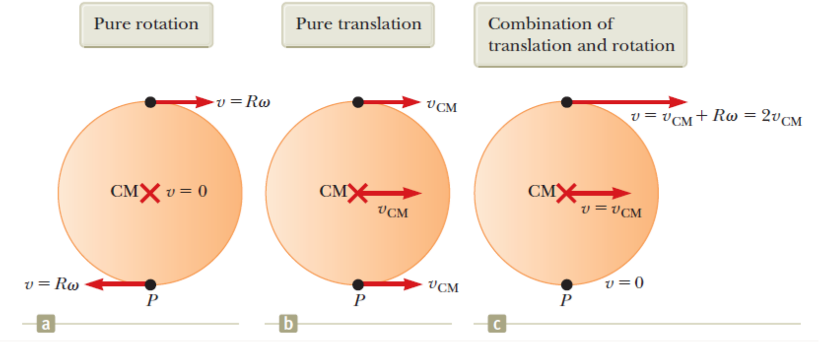


In previous pulley problems, we treated the pulley as being massless. This is a reasonable approximation whenever the weights involved are always substantially larger than the mass of the pulley, but in general the pulley's mass will have to be accounted for. In this problem, a wheel of radius 0.30 meters and mass 20 kg supports a 40 kg object. Find the acceleration of the block in two scenarios: one in which we ignore the pulley and another in which we do not. Treat the pulley as a thin disk with $\mathrm{I} = \frac{\mathrm{MR}^2}{4}$

Rolling objects



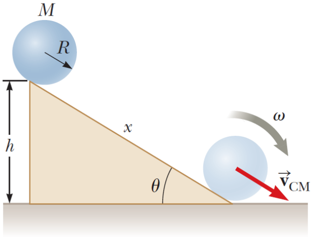
Rolling objects



The kinetic energy of a rolling wheel (non-slip) is:

$$K = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} M v_{cm}^2$$

Example 5

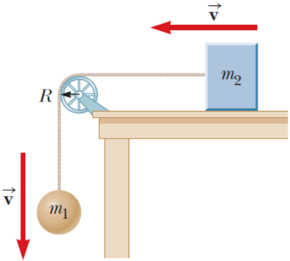


A ball of radius 0.50 meters and mass 10.0 kg roles down a ramp of height 5 meters. What is the final speed of the ball? Keep in mind that "rolling friction" does not take energy out of the system, and assume that no slipping occurs (in which case energy would be lost). For a sphere, $I = \frac{1}{3}MR^2$.

Angular Momentum

$$L = r \times P$$

Example 6



Find the total angular momentum of this system in terms of the given variables. The mass of the rim of the pulley is M, and you can consider the rest of the pulley to be virtually massless. Also find an expression for the linear acceleration of the two masses.