SOLUTIONS TO SIGNIFICANT DIGITS (FIGURES) WORKSHEET

ABSTRACT. When we make measurements, we can never achieve infinite precision. The measurements that we report must reflect those limitations. If we perform calculations using those numbers, the resulting calculation must also express those limitations.

PROBLEM 1

$2.30m \times 0.0055m =?$ Look at the numbers being multiplied. $2.30m$ has three sig figs (decimal is present so we start counting from left to right) and $0.0055m$ has two sig figs (decimal is present here as well, but we only count numbers after we get past all of the zeros). Thus the final answer should have the smallest of those two sig fig numbers, two.

> The rule for multiplication and division is that the smallest number of sig figs in the calculation (think of it as the weakest link in the chain) determines how many sig figs go in your answer.

When we multiply those numbers in a calculator, we get 0.01265. Since the decimal is present we count from left to right, but we don't start counting until we get past the zeros, thus that number has four sig figs. We need to round it to have two sig figs. We first underline the two leftmost digits: 0.01265. Next we look at the number just right of the last underlined digit. If that number is five or greater, then we round up. In this case, the number is 6, so we round up the two to a three, and our answer is 0.013. **Almost!**. Actually, we can't forget the units! When we multiple two units together, they become squared just like a number, so that $m \times m = m^2$, and our answer should be $0.013m^2$.

PROBLEM 2

$0.04030m \div 2.04s =?$. We will ignore the units at first and just focus on the numbers. 0.04030 has four sig figs (decimal is present, count left to right but don't start counting until we hit a non-zero number). 2.04 has three sig figs. Thus our answer should have the smaller of the two–should have three sig figs. When we do the division in our calculator we get 0.019754902, which is way too many!

The first thing we do is underline the three left most significant digits: 0.019754902. Notice how we don't underline the first two zeroes as they are not significant. We look at the number to the right of the last underlined number. Is it greater than or equal to 5? If not, then we don't change the number, if so, then we round up. In this case, it is 5, so we round the 7 up to a 8, and our answer is 0.0198. **Not quite.** We can't forget our units: $0.0198m/s$.

PROBLEM 3

$8010cm \times 2683cm = 21490830cm^2$, but since the smallest number of sig figs is three (8010 has no decimal so we count from right to left, but we don't count the first zero, so it has three sig figs), our answer has to be rounded to $21500000cm^2$.

PROBLEM 4

$438.3 inches \div 200 years = 2.2915 inches/year$, however, the smallest number of sig figs is....well this one is slightly ambiguous. If we know that something happened in precisely 200 years, then that number has three sig figs. Unless told otherwise, please don't assume such precision, so that 200 has one sig-fig and our answer should be $2 inches/year$.

PROBLEM 5

$45.92 km \times 0.230 km$ =? 45.92 has four sig figs, 0.230 has three (decimal present, count from left to right, don't count until first nonzero number), so our answer should have three. Our calculator gives 10.5616, so we have to round. We write the number and underline the first three sig figs: 10.5616 and note that the number to the right of that is six, which means we round up to 10.6. **Don't forget the units!** $10.6 km^2$.

Addition and Subtraction

The rule for addition and subtraction is more straight forward. In this case, if we are adding or subtracting two measurements, the answer should have the same number of decimal places as that of the measurement with the smallest number of decimal places.

PROBLEM 6

$2.30m + 0.0055m$ =? Our calculator gives 2.3055, but 2.30 has two decimal places and 0.0055 has four, so we have to use the smaller amount, two decimal places, in our answer to get 2.30, right? **Not quite.** Actually, we have to worry about rounding, so we instead underline up to two decimal places: 2.3055. Look at the number to the right of that. If it is a five or higher then we round up. Since it is a five, we round up to 2.31. **Almost!** Don't forget the units: $2.31m$.

PROBLEM 7

$23.9m\text{L} - 4m\text{L}$ =? Our calculator gives 19.9, but since $4ml$ has zero decimal places, we can only have zero in our scientific answer, and we write 19.9, noting that the number to the right of the underlined number is 9 and so we round up to 20.

PROBLEM 8

$0.93m - 0.034m$ =? Our calculator gives 0.896:

$$\overbrace{0.93m}^{2 \text{ decimal places}} + \overbrace{0.034m}^{3 \text{ decimal places}} = \underbrace{0.896m}_{3 \text{ decimal places}} \overbrace{\Longrightarrow}^{\text{Science answer}} \underbrace{0.896m \rightarrow 0.90m}_{2 \text{ decimal places}}$$

PROBLEM 9

$$\overbrace{420km}^{0 \text{ decimal places}} + \overbrace{22.9km}^{1 \text{ decimal places}} = \underbrace{442.9km}_{1 \text{ decimal places}} \overset{\text{Science answer}}{\Longrightarrow} \underbrace{\underline{442}.9km \rightarrow 443km}_{0 \text{ decimal places}}$$

PROBLEM 10

$$\overbrace{39.00s}^{2 \text{ decimal places}} - \overbrace{29.7s}^{1 \text{ decimal places}} = \underbrace{9.30s}_{12 \text{ decimal places}} \overset{\text{Science answer}}{\Longrightarrow} \underbrace{\underline{9.3}0s \rightarrow 9.3s}_{0 \text{ decimal places}}$$

Extra Problems

PROBLEM 11

2,045 cm + 8cm = 2,053 cm. Since there are no decimals involved and we are adding, then no need for chopping off decimals or rounding.

PROBLEM 12

We are dividing here, so we have to worry about sig figs. $230kg \div 0.9m^3$ =? Our calculator gives 255.555555555555555555555555555555555555555555555555555555555555555555555555555555555555 but we know that can't be right because our scientific instruments don't have such precision! 230 has two sig figs (decimal absent, count from right to left, don't count until you hit a non-zero number) and 0.9 has only one (decimal is present, count from left to right, don't count until you hit a non-zero number). Thus our answer should have one sig fig, and $300kg/m^3$ is what we will write after rounding up (since the number right after the leftmost sig-fig is 5).

PROBLEM 13

$12.4m \times 1.7m$ =?. Our calculator gives 21.08 but we are multiplying and since the smallest number of sig figs here is 2, we round our answer to two sig figs. $\underline{21}.08$ shows us that our two sig figs are followed by a zero, so we don't round up and our answer is 21. **No it isn't!** Our answer is $21m^2$. *Don't forget the units.*

PROBLEM 14

$1,730kg - 87kg$ =? Our calculator gives 1643, and since we are subtracting and there are no decimals to worry about, that is also our scientific answer.

PROBLEM 15

$8394mm \times 0.040mm$ =? Our calculator gives 335.76. This is a multiplication problem so we aren't looking at the number of decimal places but instead at the number of sig figs. In this case, 8394 has four sig figs, and 0.0040 has two sig figs (if you don't see why this is so, please send me an e-mail as soon as possible!). Thus our answer should only have two sig figs, and our final step is rounding: $\underline{33}5.76$ and we round up in this case to 340 (if you don't see why this is so, please e-mail me as soon as possible!).