Physics 185F2013 Lecture Eight

Nov 19, 2013

Dr. Jones[1]

[1]Department of Physics
Drexel University

November 19, 2013

Static Equilibrium

Condition 1

$$\begin{aligned} \sum \vec{\mathrm{F}}_{\mathrm{x}} &= 0 \\ \sum \vec{\mathrm{F}}_{\mathrm{y}} &= 0 \\ \sum \vec{\mathrm{F}}_{\mathrm{z}} &= 0 \end{aligned}$$

or simply,

$$\sum \vec{\mathrm{F}} = 0$$

Condition 2

$$\sum \vec{\tau} = 0$$

In simple terms, the net forces in all directions are zero, and there is no net torque that would cause rotation.

Blueprint for problem solving

PROBLEM SOLVING Statics

1. Choose one object at a time for consideration. Make a careful **free-body diagram** by showing all the forces acting on that object and the points at which these forces act. If you aren't sure of the direction of a force, choose a direction; if the actual direction is opposite, your eventual calculation will give a result with a minus sign.
2. Choose a convenient **coordinate system**, and resolve the forces into their components.
3. Using letters to represent unknowns, write down the **equilibrium equations** for the **forces**:
$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0,$$
assuming all the forces act in a plane.
4. For the **torque equation**,
$$\Sigma \tau = 0,$$
choose any axis perpendicular to the xy plane that might make the calculation easier. (For example, you can reduce the number of unknowns in the resulting equation by choosing the axis so that one of the unknown forces acts through that axis; then this force will have zero lever arm and produce zero torque, and so won't appear in the equation.) Pay careful attention to determining the lever arm for each force correctly. Give each torque a + or − sign to indicate torque direction. For example, if torques tending to rotate the object counterclockwise are positive, then those tending to rotate it clockwise are negative.
5. **Solve** these equations for the unknowns. Three equations allow a maximum of three unknowns to be solved for. They can be forces, distances, or even angles.

Center of Mass

Center of Mass

The center of mass of a system of particles is that point which moves as though all of the system's mass were concentrated there, and all external forces were applied there.

Center of Mass

Consider two particles m_1, m_2 which are a distance d apart in space. The center of mass of this system is defined by

$$\vec{x}_{com} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1 + m_2}$$

For systems of n particles, this is

$$\vec{x}_{com} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 + m_3\vec{x}_3 + \cdots + m_n\vec{x}_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

Center of Mass

EXAMPLE 01: Find the center of mass for the following system of particles:

Particle	Mass (kg)	x(cm)	y(cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

We can treat the y and x directions separately to simplify our task. For the x direction:

$$\mathrm{x_{com}} = \frac{1.2\mathrm{kg}(0) + 2.5\mathrm{kg}(140\mathrm{cm}) + 3.4\mathrm{kg}(70\mathrm{cm})}{7.1\mathrm{kg}} = 83\mathrm{cm}$$

$$\mathrm{y_{com}} = \frac{1.2\mathrm{kg}(0) + 2.5\mathrm{kg}(0) + 3.4\mathrm{kg}(120\mathrm{cm})}{7.1\mathrm{kg}} = 58\mathrm{cm}$$

Center of Mass

Newton's Second Law for a System of Particles:

Newton's Second Law can be modified to apply to the entire system instead of each individual particle:

$$\vec{\mathrm{F}}_{\mathrm{net}} = \mathrm{M}\vec{\mathrm{a}}_{\mathrm{com}}$$

Where $\vec{\mathrm{F}}_{\mathrm{net}}$ is the net external force acting on the ball, M is the toal mass of the system, and $\vec{\mathrm{a}}_{\mathrm{com}}$ is the acceleration of the center of mass of the system.

Example for cm

Three particles are initially at rest and then experience external forces. Particle A is 4.0 kg and experiences a net force of 6.0 N in the -x direction, particle B is 8.0 kg and experiences a net force of 12 N at 45 degrees above the x axis, and positive C is 4.0 kg and experiences a net force of 14 N in the positive x direction. What is the acceleration of the center of mass and in what direction will it move?

Example for cm

The x and y components of the acceleration are:

$$a_{\mathrm{com,x}} = \frac{F_{A,x} + F_{B,x} + F_{C,x}}{M} = \frac{-6.0\mathrm{N} + 12\mathrm{N}\cos(45^\circ) + 14\mathrm{N}}{16\mathrm{kg}} 1.03\mathrm{m/s^2}$$

$$a_{\mathrm{com,y}} = \frac{F_{A,y} + F_{B,y} + F_{C,y}}{M} = \frac{0 + 12\mathrm{N}\sin(45^\circ) + 0}{16\mathrm{kg}} = 0.530\mathrm{m/s^2}$$

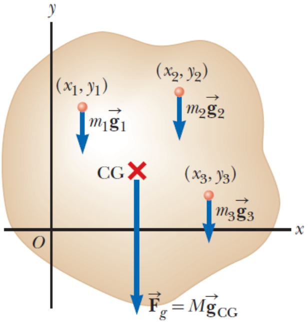
The net velocity is

$$a_{\mathrm{com}} = \sqrt{(a_{\mathrm{com,x}})^2 + (a_{\mathrm{com,y}})^2} = 1.16\mathrm{m/s^2}$$

The angle is

$$\theta = \tan^{-1}\frac{a_{\mathrm{com,y}}}{a_{\mathrm{com,x}}} = 27^\circ$$

Center of Gravity

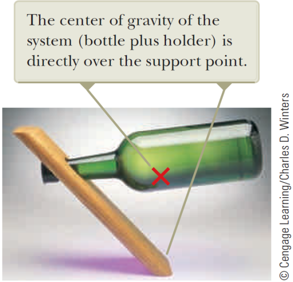


Mathematically, we can often consider a single point in a solid object rather than to consider every piece of that object.

$$\begin{aligned} U_g &= \sum_i m_i g y_i \\ M g Y_{CM} &= g \sum_i m_i y_i \\ Y_{CM} &= \frac{\sum_i m_i y_i}{M} \end{aligned}$$

and also, $X_{CM} = \frac{\sum_i m_i x_i}{M}$.

Torque and Center of Gravity



We can choose any coordinate origin and find that:

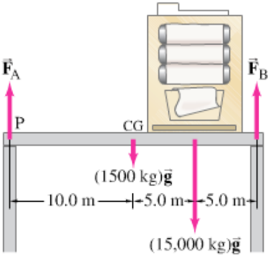
$$\begin{aligned}\vec{\tau}_{\mathrm{net}} &= \sum_{\mathrm{i}} \left(\vec{\mathrm{r}}_{\mathrm{i}} \times \vec{\mathrm{F}_{\mathrm{g,i}}}\right) \\ &= \left(\sum_{\mathrm{i}} \mathrm{m}_{\mathrm{i}}\vec{\mathrm{r}}_{\mathrm{i}}\right) \times \vec{\mathrm{g}} \\ &= \vec{\mathrm{r}}_{\mathrm{cm}} \times \mathrm{M}\vec{\mathrm{g}}\end{aligned}$$

It is best to find the most convenient origin.

Remember

Remember, when solving these problems, DON'T PANIC. Follow the systematic procedure outlined in these notes. Even if you aren't sure how to solve the problem, just start by applying the procedures and letting the physics guide your solution.

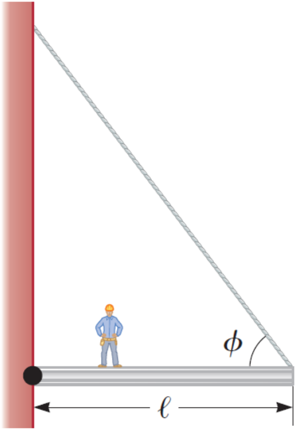
An example 0



A uniform 1500 kg beam is 20 m long and supports a 15,000 kg printing press 5 meters from the right support column. Calculate the force on each support column.

An example 0

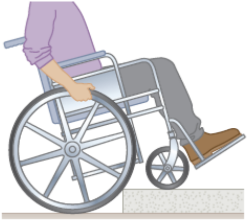
An example 1

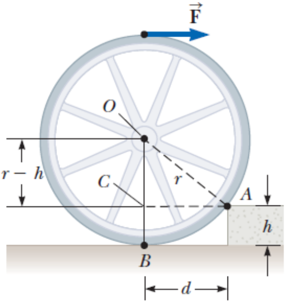


A uniform horizontal beam with length l = 8.00 m and a weight of $W_b = 200$ N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\phi = 53.0°$ with the beam. A person of weight $W_p = 600$ N stands distance d = 2.00m from the wall. Find the tension in the cable and the forces exerted by the wall onto the beam.

An example 1

An example 2





Estimate the magnitude of the force needed for the wheel-chair to roll up over the sidewalk curb. Your answer should be in variables.

An example 2