Name: ______________________________

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. *Please make an effort on all problems, partial credit is awarded for effort-based solutions which demonstrate familiarity with the physics concepts.*

1. (5 points) An ice skater spins at 2.5 rev/s when her arms are extended. She draws her arms in and spins at 6.0 rev/s. By what factor does her intertia change in the process. *Hint: Conservation of angular momentum.*

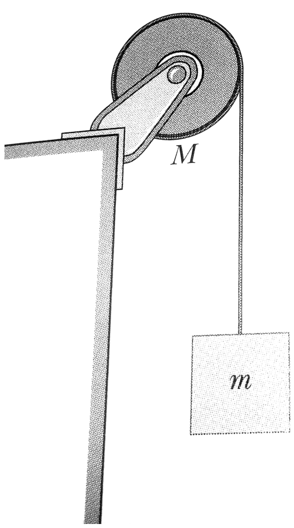
Solution:

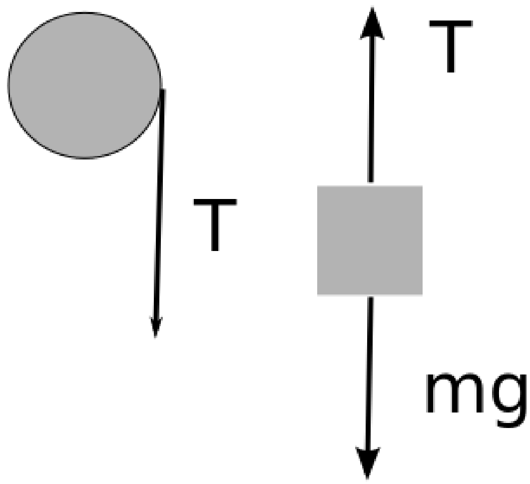
$$I_1\omega_1 = I_2\omega_2 \quad \rightarrow \quad \frac{I_2}{I_1} = \frac{\omega_1}{\omega_2} = \frac{2.5}{6.0} = 0.42$$

2. (2 points) Newton's second law in angular form is (*Hint: Each of these equations will be useful on this test*).

○ $\sum F_x = ma$ ○ $\sum F_y = ma$ ○ $I_1\omega_1 = I_2\omega_2$ √ $\sum \tau = I\alpha$

3. (10 points) A uniform disk with mass $M = 3.0$ kg and radius $R = 10cm$ is mounted on a fixed horizontal axle. A block with mass $m = 2.0$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block. *Hint: The angular intertia of a disk is $\frac{1}{2}MR^2$. What is the net torque on the disk? Is the angular acceleration α zero or not?*





Solution: For the block,

$$\sum F_y = T - mg = ma$$

For the disk,

$$\sum \tau = -RT = I\alpha$$

$I = \frac{1}{2}MR^2$, so this equation becomes:

$$\sum \tau = -RT = \frac{1}{2}MR^2\alpha \quad \rightarrow T = \frac{1}{2}MR\alpha$$

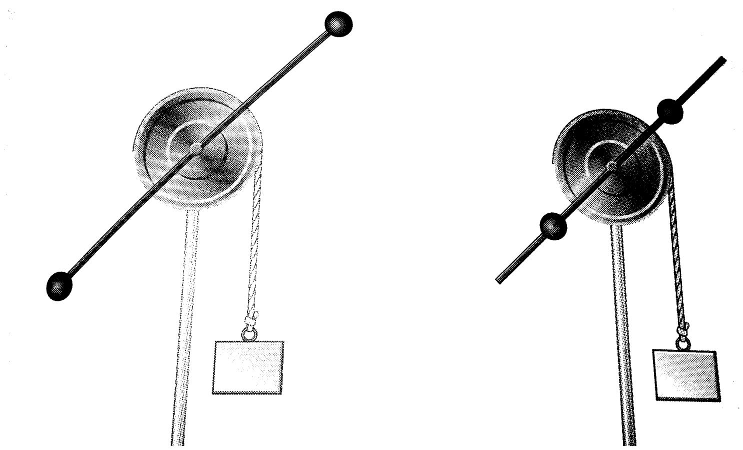
Since the taut cord is wrapped around the very edge of the disk, we know that the acceleration of the block will be $a = \alpha R$ such that $\alpha = a/R$ and,

$$T = -\frac{1}{2}Ma$$

Then our force 2nd Law equation becomes,

$$-\frac{1}{2}Ma - mg = ma \quad \rightarrow \quad a\left(m + \frac{M}{2}\right) = -mg \quad \rightarrow \quad a = -\frac{mg}{m + \frac{M}{2}} = 5.6\,\frac{\text{m}}{\text{s}^2}$$

4. (3 points) Which of the following two systems (left or right) will spin the fastest and why? The solid disk, spheres, and box all have the exact same mass, the only difference is the location of the spheres which alters the intertia. *Hint: You don't need to solve for inertia, or do much solving at all. Think of Newton's Second Law in angular form. Which of the two systems will have the greater angular intertia?*



Solution:

Newton's second law in angular form tells us that

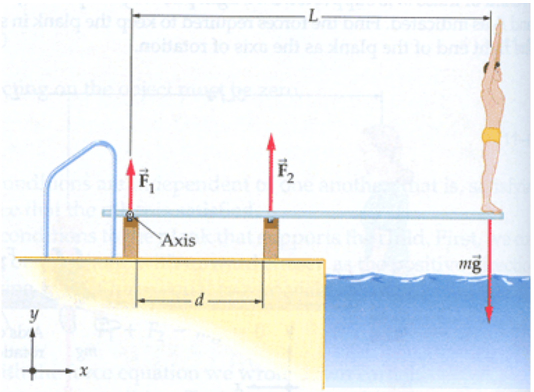
$$\sum \tau = I\alpha$$

The angular inertia of the system on the left will be greater since it is proportional to how far away the spheres are (mr^2). Since the torque is the same in both cases, then the angular acceleration of the *left* system must be *less than* that of the right system since its angular inertia is greater.

For the same applied force, a greater angular inertia results in a slower angular acceleration.

5. (5 points) (Bonus) List three important things you've learned from physics, and provide a real world example of each concept.

6. (15 points) A 5.00 m long diving board of negligible mass is supported by two pillars. One pillar is at the left end of the diving board, as shown, and the other is 1.50 m away. Find the forces exerted by the pillars when a 90.0 kg diver stands at the far end of the board. *Hint: F_1 in the graph will turn out to be negative. Why?*



Solution:

The forces shown on the graph are our first guess. With force diagrams, if you guess the wrong direction, the mathematics will tell you so by giving you a negative number for the magnitude in your solution, which is indeed what happens here.

Newton's Second Law in linear form gives:

$$\sum F_Y = F_1 + F_2 - mg = 0$$

Newton's Second Law in angular form gives:

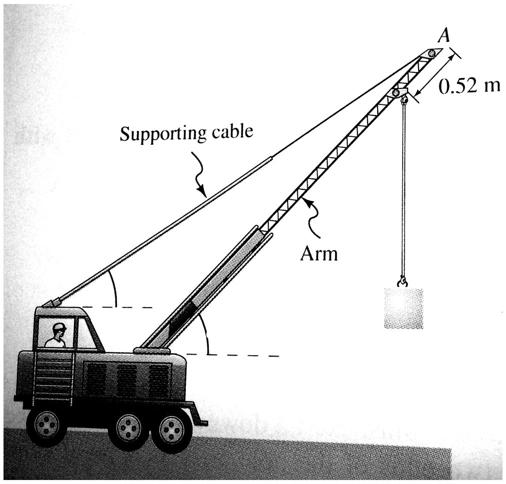
$$\sum \tau = F_2 d - mgL = 0 \quad \rightarrow \quad F_2 = \frac{mgL}{d} = 2940N$$

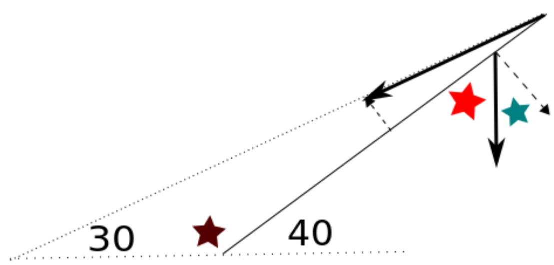
We plug this into the first equation:

$$F_1 + \frac{mgL}{d} - mg = 0 \quad \rightarrow \quad F_1 = mg - \frac{mgL}{d} = -2060N$$

The negative sign means that F_1 actually points in the downward direction, contrary to what we assumed. This demonstrates the flexibility of Newton's second law equations for finding both the magnitude and direction of a vector.

7. (15 points) A massive crane (assume it is fixed to the Earth) is lifting a mass of 7600 kg. The arm of the crane is supported at its base at point B by a strong pivot and at its top at point A by a cable. The arm makes an angle with the horizontal of 40° and the cable makes an angle 30° with the horizontal as well. The arm is 12.0 m long. The mass is lifted from a point on the arm 0.52 m from the end point A. Assume that the mass of the arm is small enough to ignore, find the tension in the cable.





Solution:

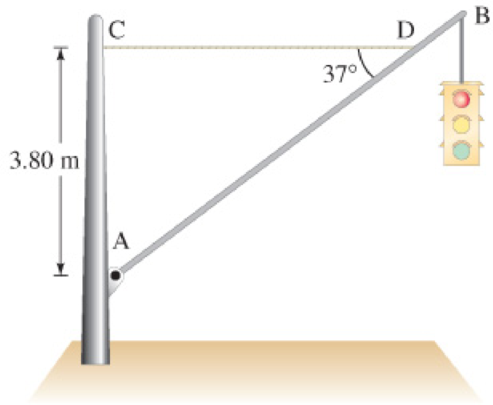
There will be two forces at the base of the arm, one horizontal and one vertical, but we leave those out in our graph above since we are only be asked for the tension and we can apply Newton's 2nd Law for angular form. When doing so, we get the following equation:

$$\sum \tau = 0 = T\sin(10)(12.0m) - (7600kg)(9.8\frac{m}{s^2})\cos(40)(12-0.52)m$$

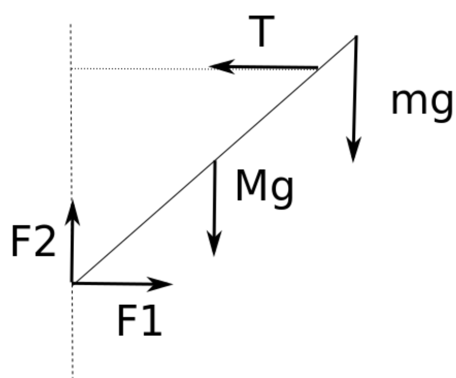
We can solve this for T to find that $T = 314328.7N$. A note about the angles, we recall that all the angles in a triangle add up to 180, and that there is 180 degrees in a full line (half circle) so that the purple star in the figure is 180-40 = 140 degrees. The sum of the purple star and the angle the cable makes is 140 + 30 = 170, so there remains 10 degrees left, which is the angle that the cable makes with the arm. The red star is 90-40 = 50, but the blue star is 90-50 = 40 (compliments of a right angle).

8. (15 points) A traffic light hangs from a pole as shown in the figure. The uniform aluminum pole AB is 7.20 m long and has a mass of 11.0 kg. The mass of the traffic light is 20.5 kg.

- Determine the tension in the horizontal massless cable CD.
- Determine the vertical component of the force exerted by the pivot A on the aluminum pole.
- Determine the horizontal component of the force exerted by the pivot A on the aluminum pole.



Solution:



First things are first: The tension from the cable *is not* acting at the tip of the pole, but only partially up the pole and before the hanging stop light. The distance that the cable connects to the hanging pole is the hypotenuese of a triangle which has a side of 3.8 m and opposite angle of 37 degrees, so the length of this hypotenuse is

$$l \sin(37) = 3.80m \quad \rightarrow \quad l = \frac{3.80m}{\sin(37)} = 6.3m$$

If we start by considering Newton's 2nd Law for angular rotation, and take our rotation through the base A, then we get

$$\sum \tau = T \sin(37)(6.3m) - (20.5kg)(g)\cos(53)(7.2m) - (11.0kg)(g)\cos(53)(7.2m/2) = 0$$

We show the force of the weight of the pole acting from its center of mass (middle) since it is *uniform*. We can solve this T to find that $T = 291.2N$

Next we sum the forces in the x y directions, and since the system is static, these sums are zero, in which case:

$$\sum F_x = F_1 - T = 0 \quad \rightarrow \quad F_1 = T = 291.2N$$

and

$$\sum F_y = F_2 - Mg - mg \quad \rightarrow \quad F_2 = (M + m)g = 308.7N$$

9. (3 points) List at least one of Newton's laws.

> **Solution:** See notes / books.

10. (1 point) Describe the difference between elastic and inelastic collisions.

> **Solution:** Elastic–energy conserved. Inelastic–energy not conserved.

11. (3 points) An old spaceship has a rotating tape recording device for storing data. What happens to the spaceship when the tape starts spinning? Why? *Hint: Something is being conserved.*

> **Solution:** It spins slightly in the opposite direction to preserve angular momentum for the entire system.

12. (3 points) Why are the curved ramps on and off major highways banked at an angle rather than kept horizontal?

> **Solution:** A centripetal force is needed to keep something going in a circular path. The car relies on the friction between its wheels and the road to provide this force (directed towards the center of the circle), but for fast turns this friction may not be enough and the car could slide (especially if road conditions are bad such as rain/snow/ice). By banking the road, engineers have used gravity to ensure that the car will get an extra counter force by the component of gravity directed down the ramp.