Name: ____________________________

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. *Please make an effort on all problems, partial credit is awarded for effort-based solutions which demonstrate familiarity with the physics concepts.*

1. Calculate the potential kinetic energy of a solid ball of mass 2 kg and radius 0.20 m that rolls without slipping on a flat surface at a speed of 0.8 m/s. *Hint: Account for both translational and rotational kinetic energy. For a solid sphere, $I = \frac{2}{5}mr^2$.*

Solution:

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Substituting what we are given for I:

$$K = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2}mv^2 = \frac{1}{5}mr^2\omega^2 + \frac{1}{2}mv^2$$

Now for a rolling ball, we know that the speed of the ball is equal to the radius of the ball times the angular speed (every full rotation of 2π lays out a full circumference worth of movement) so that $v = r\omega$, and we can write the above equation as

$$\begin{aligned} K &= \frac{1}{5}mr^2\omega^2 + \frac{1}{2}mv^2 \\ &= \frac{1}{5}mv^2 + \frac{1}{2}mv^2 \\ &= \frac{7}{10}mv^2 \\ &= (0.7)(2\text{ kg})(0.8\frac{\text{m}}{\text{s}^2})^2 \\ &= 0.9\text{ J} \end{aligned}$$

2. Two equal heavy masses are positioned at either end of a rod with negligible mass. The rod is mounted in the center in such a way that it can spin freely, the masses are each a distance r_1 away from the center of spinning, and it is set to spin at a speed of ω_1. What will be the rotational speed (ω_2) of the setup if the masses are moved half-way inward (so that $r_2 = r_1/2$)? *Hint: Assume we can treat the masses as point particles such the inertia is $I = mr^2 + mr^2 = 2mr^2$, and recall that angular momentum is equal to $L = I\omega$.*

Solution: The key to this problem is assuming conservation of angular momentum:

$$L_1 = L_2$$

where $L_1 = I_1\omega_1$ and $L_2 = I_2\omega_2$. Now $I_1 = 2mr^2$ and

$$I_2 = 2m\left(\frac{r}{2}\right)^2 = \frac{mr^2}{2}$$

The conservation of angular momentum condition gives:

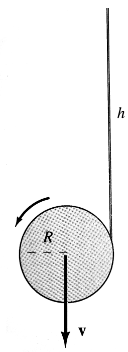
$$2mr^2\omega_1 = \frac{mr^2}{2}\omega_2$$

Then

$$\omega_2 = \frac{4mr^2\omega_1}{mr^2} = 4\omega_1$$

Thus by reducing the distance of the masses from the center of the spinning by one half, the angular speed is four times larger.

3. Show that the velocity of an unwinding spool of mass m, angular inertia I and radius r is equal to $v = r\sqrt{\frac{2mgh}{mr^2+I}}$ after it has fallen a height h.



Hint: This isn't as tedious to solve as it might seem at first!

- *Step 1: Find the Kinetic Energy of the unwinding spool and remember that you need to account for both linear and angular kinetic energy;*
- *Step 2: Use conservation of mechanical energy to relate the height that the spool has fallen to the change in its total kinetic energy. Since the spool is unwinding at the rightmost tip, then we can relate $v = r\omega$.*
- *Step 3: Solve for v.*

Solution: The kinetic energy for the system is

$$KE = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Using the relation $v = r\omega$, we find that

$$KE = \frac{1}{2}I\omega^2 + \frac{1}{2}mr^2\omega^2$$

$$KE = \frac{1}{2}(I + mr^2)\omega^2$$

By conservation of mechanical energy, the kinetic energy of the spool after it has fallen a height h is $KE = PE = mgh$, so that

$$\frac{1}{2}(I + mr^2)\omega^2 = mgh$$

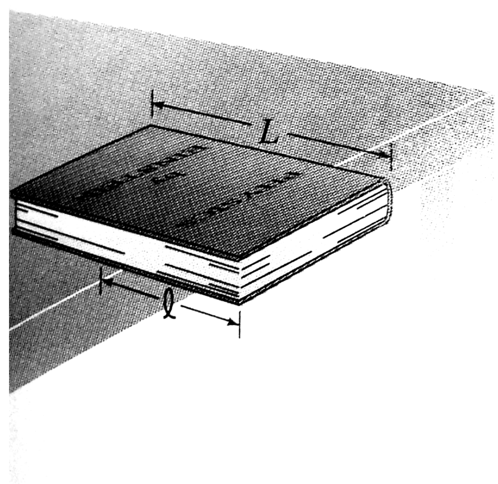
Now we can solve for ω:

$$\omega = \sqrt{\frac{2mgh}{mr^2 + I}}$$

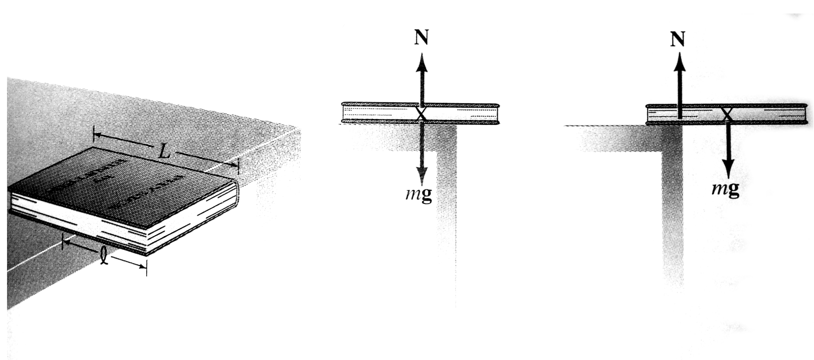
Finally, since $v = r\omega$,

$$v = r\sqrt{\frac{2mgh}{mr^2 + I}}$$

4. Treating a book as a rectangular box with uniform density and length L, lying with an edge parallel to a table and also hanging off the table's edge by an amount l, what is the largest possible value of l for which the book does *not* rotate off the edge? *Hint: You can treat gravity as acting from the center of mass.*

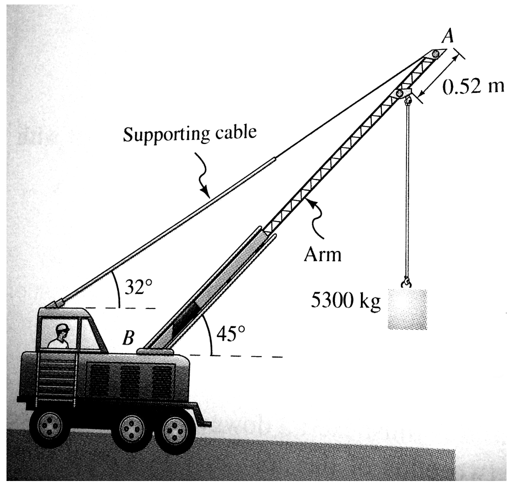


Solution: Recall that static equilibrium requires the balances of all forces **and** the balance of all torques.

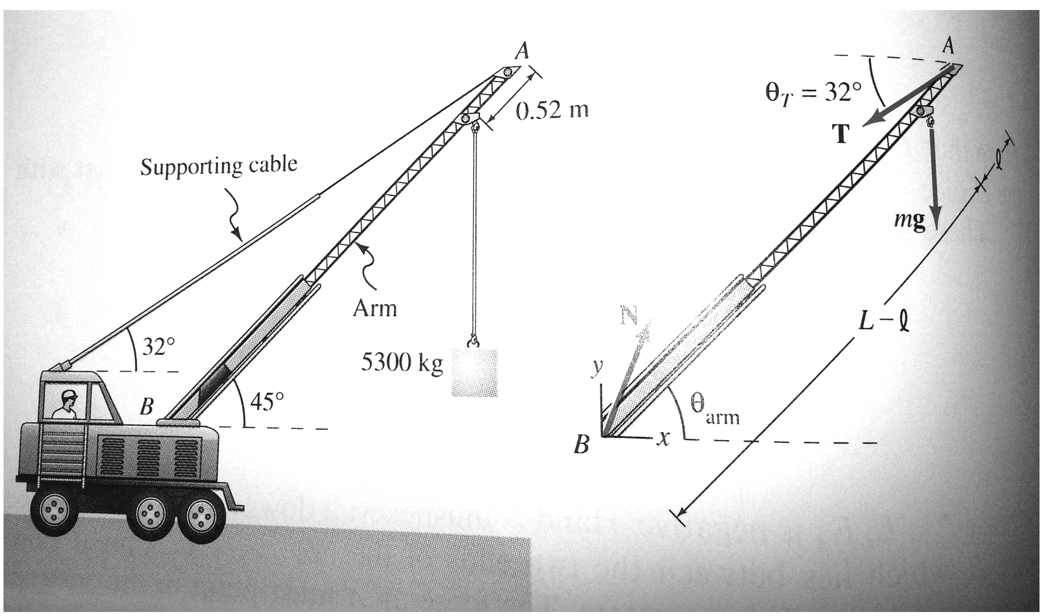


Once the center of mass crosses the edge of the table, the net torque on the book (as seen from a point of rotation at the edge of the table) becomes non-zero and the book will rotate clockwise. (Your answer should include a figure as above).

5. A massive crane (assume it is fixed to the Earth) is lifting a mass of 5300 kg. The arm of the crane is supported at its base at point B by a strong pivot and at its top at point A by a cable. The arm makes an angle with the horizontal of 45° and the cable makes an angle 32°. The arm is 10.0 m long. The mass is lifted from a point on the arm 0.52 m from the end point A. Assume that the mass of the arm is small enough to ignore, find the tension in the cable. *Hint: the angle the tension cable makes with the arm of the crane is* 13°



Solution:



If the crane is in static equilibrium, then the net torque equation (along with the net force equation) will be zero. We must decompose the tension and weight vectors into components perpendicular to the arm. Also, we can eliminate the unknown normal force by taking our pivot point as being the point B, so that

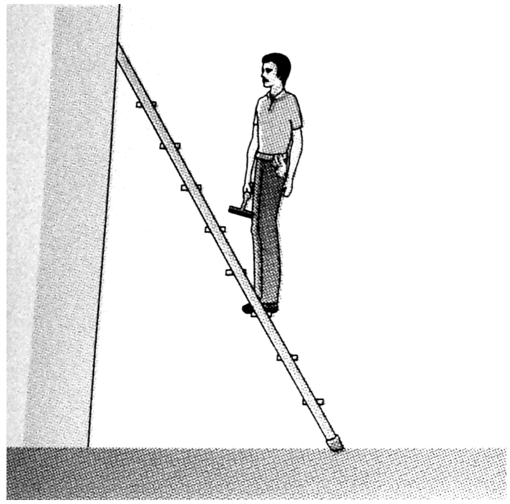
$$\begin{aligned}\sum \tau &= \\ &= -(L-l)mg\cos(45) + LT\sin(13) \\ &= 0\end{aligned}$$

We can thus solve for T:

$$LT\sin(13) = (L-l)mg\cos(45) \quad \rightarrow \quad T = \frac{(L-l)mg\cos(45)}{L\sin(13)} = 154777.3 \text{ N}$$

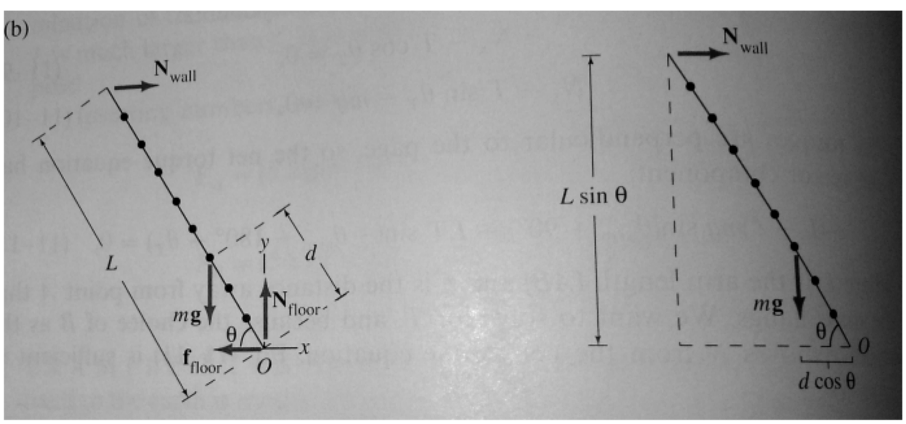
Which we would write as 1.5×10^3 N with sig figs (you won't be tested on sig figs).

6. A ladder of length 3.0 m with eight rungs spaced 0.33 m apart is leaning against a wall at an angle of 58°. Ignore the ladder's mass. A window washer of mass $m = 85kg$ is climbing the ladder. The coefficient of static friction between the rubber feet of the ladder and the floor is $\mu_s = 0.51$, but assume that the wall is smooth and frictionless. Is the ladder safe for climbing for the washer as he ascends to the seventh rung of the ladder?



Solution:

Solution: Recall that the **maximum** value that static friction can take on is $f = \mu_s N$. The force of friction is equal and opposite to the component of the applied force along a surface up until it reaches that maximum. We will first solve the problem in general to find the friction force, and then in particular to find out if we have exceeded the maximum friction force at the seventh rung.



Using the labels shown in the figure above, and applying the net force equations for static equilibrium (**Newton's Second Law**). We first get the following two equations:

$$N_{floor} - mg = 0$$

$$N_{wall} - f_{floor} = 0$$

It is clear that we need to find N_{wall} in order to determine the friction force from the floor. We take our net torque equation such that our center of rotation is where the floor meets the ladder (to eliminate two unknowns at once) to get the following equation:

$$\begin{aligned} \sum \tau &= \\ &= mgd\cos(\theta) - N_{wall}L\sin(theta) \\ &= 0 \end{aligned}$$

where d is the length of the worker up the ladder, and we have taken the perpendicular components of the radius relative to the forces. The signs of the above equation comes from the convention that a torque that would cause a clockwise rotation will have a negative sign. From the net force equation, we have that $N_{wall} = f_{floor}$ so that,

$$f_{floor} = \frac{mgd\cos(\theta)}{L\sin(\theta)}$$

We calculate that seven rungs up the ladder corresponds to $d = 2.33$ m, $\theta = 58°$, so

$$f_{floor} = \frac{(85 \text{ kg})(9.8\frac{\text{m}}{\text{s}^2}(2.33m)\cos(58°)}{3.0 \text{ m}\sin(58°)} = 404.27N$$

We also have from the net force equations that $N_{floor} = mg = 833N$ so that the maximum friction force that could be exerted by the floor to hold up the ladder would be $\mu_S mg = 424N$. At the seventh rung, we are just below that amount so the climber is safe, but if he climbs to the next rung, the ladder will slide out and crash to the floor as it will exceed the maximum static friction threshold.