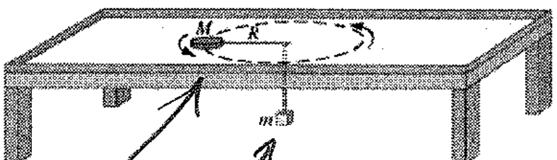
Name: ______________________________

> You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

1. **Modified version of Chapter 5: 80**

> A small object of mass M moves in a circular path of radius R on a frictionless horizontal table. Iti s attached to a string that passes through a small frictionless hole in the center of the table. A second object with mass m is attached to the other end of the string. Show that the speed of the object on the table is
>
> $$v = \sqrt{\frac{mgR}{M}}$$



Two ways to do freebody diagram:

a_c Net centripetal acceleration

T

Table Frame

F T

Reactive Centrifugal

Puck's Frame

Either way gets the right answer.

T

mg

$$\Sigma F_y = 0 = T - mg \Rightarrow T = mg$$

a

T

$$\Sigma F_r = T = Ma_c$$

$$mg = Ma_c \rightarrow mg = M\frac{v^2}{R}$$

$$\rightarrow V = \sqrt{\frac{mgR}{M}}$$

Table frame, non-fictio

F_c T

$$F_c = T$$

$$\frac{Mv^2}{R} = mg \rightarrow V = \sqrt{\frac{mgR}{M}}$$

Puck's Frame

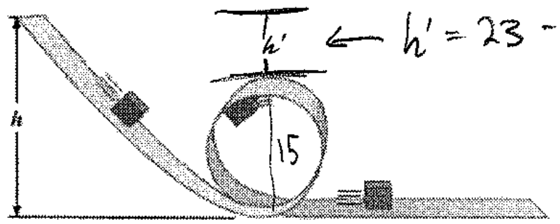
Same answer

Please go on to the next page…

2. **Chapter 7: 47**

> A 1500 kg roller coaster car starts from rest at a height h=23.0 m above the bottom of a 15.0 m diameter loop. If friction is negligible, determine the downward force (normal force) of the rails on the car when the upside down car is at the very top of the loop. **Also**, what minimum height h would allow the car to stay on the loop at the very top? In other words, how small can *h* be until the car doesn't make it around the loop as it did in class? *Hint: The crucial point is when the net force holding the car up goes to zero.*

23m



$h' = 23 - 15 = 8\,m$

(Part 1)

$\Delta E_{mech} = 0$

(No friction)

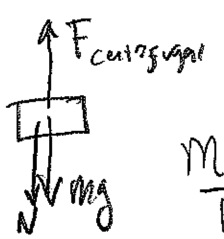
$U_{g_i} + K_i = U_{g_f} + K_f$

$\cancel{m}g(23) + 0 = \cancel{m}g(15) + \frac{1}{2}\cancel{m}v^2$

$\sqrt{2(g(8))} = V$

$g = 9.8$ $\quad V = 12.5\,\frac{m}{s}$

Part 2



$\frac{mV^2}{R} = F_N - mg \rightarrow F_N = \frac{mV^2}{R} - mg$

$\frac{1500\,(12.5)^2}{(15/2)} - 1500(9.8) = F_N = 16{,}600\,N$

① Free body
② Newton's Second Law
③ Solve

3. Chapter 9: 78

Two objects of mass $m_1 = 500$ g and $m_2 = 510$ g are connected by an ideal string of negligible mass that passes over a pulley with frictionless bearings but has mass 50.0 g and radius 4.00 cm. Treat the pulley as a disk. *Hint: Because there are two forces acting on a non-ideal pulley, unlike our problem with a block on the table, in this case the tension is not the same on either side of the pulley.*

(a) Find the acceleration of the objects.

(b) What is the tension in the string between the 500 g block and the pulley?

(c) What is the tension in the string between the 510 g block and the pulley?

(d) Previously we said that the tension in a string is uniform. That was a bit over-simplistic. Now that we are dealing with realistic pulleys, we have to account for the fact that the pulley has mass and that the point of contact between the pulley and rope is one of static friction just like the point of contact between a wheel and the road is static friction as discussed in class. Examples 9-12 and 9-13 will help you with this problem. In a sentence or two, explain why we can no longer assume that the tension in a rope is uniform throughout when we are talking about non-idealistic pulleys.

$I = \frac{1}{2}MR^2$

$F_{T1} = M_1(g+a)$
$= (.500)(9.9)$
$= 4.95\,N$

$F_{T2} = M_1(g-a)$
$= 0.51(9.7)$
$= 4.947$
$\sim 4.95\,N$

The inertia of the pulley adds to the tension on one side and detracts from the tension on the other side.

Newton's Second Law

① $\sum F_{y_1} = F_{T_1} - M_1 g = M_1 a$

② $\sum F_{y_2} = F_{T2} - M_2 g = M_2(-a)$

③ $\sum \tau = F_{T2}R - F_{T1}R = I\alpha$

$S = \theta R \to V = \omega R \to a = \alpha R$
$\to \alpha = \frac{a}{R}$

$\to F_{T2}R - F_{T1}R = I\frac{a}{R} = \frac{1}{2}MR^2\frac{a}{R}$
$= \frac{MRa}{2}$

divide out R

$F_{T2} - F_{T1} = M\frac{a}{2}$

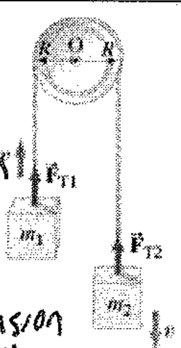
From ①,②

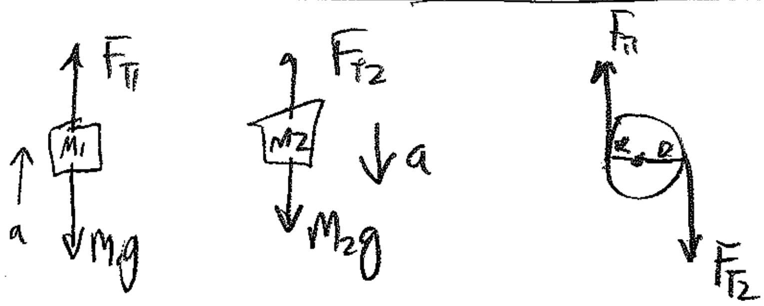
$F_{T1} = m_1(a+g)$
$F_{T2} = M_2(g-a)$

$M_2(g-a) - M_1(g+a) = M\frac{a}{2}$
$M_2 g - M_2 a - M_1 g - M_1 a = Ma/2$

$(M_2 g - M_1 g) = \left(\frac{M}{2} + M_2 + M_1\right)a$

$a = \frac{(M_2 - M_1)g}{\left(\frac{M}{2} + M_2 + M_1\right)}$
$= \frac{(.01)(9.8)}{\left(\frac{M}{2} + M_2 + M_1\right)}$
$= 0.095\,m/s^2$
$= 0.1\,m/s^2$

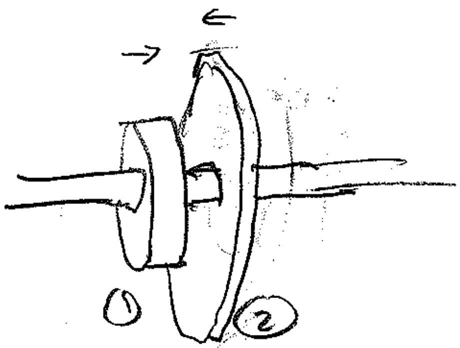




Free Body

4. **Chapter 10:52**

   Two disks of identical mass but different radii r and $2r$ are spinning on frictionless bearings at the same angular speed ω_0, but in opposite directions. They are slowly brought together and frictional force between the surfaces brings them to a common angular velocity. What is the magnitude of that final angular velocity in terms of ω_0? What is the change in rotational kinetic energy of the system? Explain.



Angular momentum is conserved.

Angular KE is not.

I for disk is: $I = \frac{1}{2}MR^2 \rightarrow$ thin

(when rotated around center)

$$L_i = L_f$$

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

Same speed $\longrightarrow$ opposite direction

$$I_1\omega_0 - I_2\omega_0 = (I_1 + I_2)\omega$$

$$(I_1 - I_2)\omega_0 = I_1 + I_2\,\omega$$

$$\omega = \frac{I_1 - I_2}{I_1 + I_2}\omega_0$$

$$K_i = \frac{I_1}{2}\omega_0^2 + \frac{I_2}{2}\omega_0^2$$

$$= \frac{1}{2}(I_1 + I_2)\omega_0^2$$

$$K_f = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1 - I_2}{I_1 + I_2}\omega_0^2\right)$$

$$= \frac{1}{2}\left(\frac{(I_1 - I_2)^2}{I_1 + I_2}\right)\omega_0^2 \longrightarrow$$ This is less than K_i

5. **Not from book** A cyclist accelerates from rest at a rate of 0.07 $\frac{\text{m}}{\text{s}^2}$. The tires have a diameter of 60 cm and assume an entire wheel weighs around 2.50 kg (includes rim).

$R = 0.3\ \text{m}$

$V = r\omega$

(a) How fast will a point on the rim of the tire at the very top be moving after 3.0 s? *Hint:*

$V_f = \cancel{V_i} + at$

$$v_{\text{top rel ground}} = v_{\text{top relative to center}} + v_{\text{center relative to ground}}$$

$V_f = 0.07t$

$= V + V = 2V = 2(0.07\,\text{m/s}^2)(3)$

$= 0.42\,\text{m/s}$

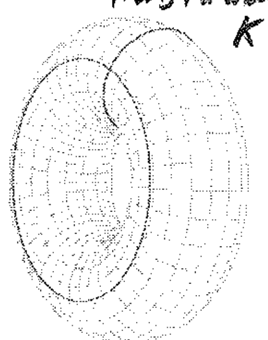
(b) Give the wheel's kinetic energy as a function of time. What power must be generated for this wheel? Treat the wheel as a thin ring with spokes of negligible mass so that $I = MR^2$ and recall that the no slip condition is that $v = \omega R$.

$V = r\omega \Rightarrow \omega = V/R = \left(\frac{0.07t}{0.3\,\text{m}}\right)$

Rotational K: $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(MR^2)\omega^2 = \frac{1}{2}MR^2\frac{V^2}{R^2}$

$= \frac{1}{2}MV^2$

Translational K: $K_L = \frac{1}{2}MV^2$



$K_{TOT} = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = K_R + K_L = MV^2$

$= M(0.07t)^2 = (2.5)(0.07)^2t^2$

$\longrightarrow P = \frac{dK}{dt} = 0.0245\,t$

(c) Let's get even more realistic. The thin-ring approximation for a bike wheel isn't a great approximation. A more fitting approximation might be the moment of inertia of a torus (a doughnut shaped object that is hallow on the inside). For a torus of large radius $a = 30cm$ (for the bike in this problem) and small radius (the radius of a cross section of the tube of the tire) $b = 2cm$, the moment of inertia is given as:

$$I = \left(a^2 + \frac{3}{4}b^2\right)m$$

Recalculate the kinetic energy of the wheel with this new value. Is there a huge difference? Was our initial approximation justified?

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(a^2+\tfrac{3}{4}b^2)M\left(\frac{V^2}{R^2}\right) = \frac{1}{2}(R^2+\tfrac{3}{4}b^2)M\left(\frac{V^2}{R^2}\right)$$

$$= \frac{1}{2}\left(1+\tfrac{3}{4}\frac{b^2}{R^2}\right)MV^2$$

$$K_{TOT} = K_R + K_L = \frac{1}{2}(2.0033\,MV^2) \qquad = \frac{1}{2}(1.0033)MV^2$$

Justified

(d) From the power you ~~calculated~~, multiply by two to apply to both wheels. Ignoring the power needed to move the rider and the rest of the bike, how much power is needed to accelerate the wheels? 1 dietary calorie is equal to approximately 4.1868×10^3 Joules. Create a function that describes the calories burned per second by the cyclist just to accelerate the wheels. How many calories would they burn in one hour if they kept accelerating?

$$0.05t \text{ Watts} = 0.05t\,\frac{J}{s}\left(\frac{1\text{ kcal}}{4.1868\times10^3\,J}\right)$$

$$= 1.17\times10^{-5}\,\frac{\text{kcal}}{s}$$

in one hour = 3600 seconds: $E = 1.17\times10^{-5}\,\frac{\text{kcal}}{s}(3600s)$
$= 0.042$

Add in the weight of the bike and the rider. Make reasonable estimates of their weight and calculate their kinetic energies as a function of time. Add up all of the kinetic energies as a function of time. Take the derivative with respect to time to find the power needed. Convert this into calories per second. How long would the rider need to accelerate to burn off a 250 calorie soft drink (again, these are dietary calories which you see on the labels of your food which are 1000 physics/chemistry calories). ~~*By the way, the acceleration in this problem is approximately equivalent to the acceleration a cyclist would need to keep the bike at a constant speed given friction in the gears/wheels of a real-world bike.*~~ If your numbers don't seem to make sense, where do you think things went wrong?

Bad wording

Bike frame = 15 kg
Rider = 81 kg
→ 96 kg

$$K_{BR} = \frac{1}{2}MV^2 = \frac{1}{2}(96)(0.07)^2t^2$$

$$= 0.2352\,t^2$$

$$\frac{dK_{BR}}{dt} = 0.47t\,\frac{J}{s} = 0.47t\,\frac{J}{s}\left(\frac{1\text{ kcal}}{4.1868\times10^3}\right) = 1.12\times10^{-4}\,t\,\frac{\text{kcal}}{s}$$

End of exam

$$\int_0^{t'} P(t) = 250\text{ kcal} = \int_0^{t'} (1.12\times10^{-4}\,t + 1.17\times10^{-5}t)$$

$$= \frac{1.237\times10^{-4}}{2}t'^2 \rightarrow t' = \text{half hour}$$

Not realistic, Friction Requires Constant power