Name: ________________________________

You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

Short Problems

$$
\begin{aligned}
v_f &= v_i + at \\
x_f &= x_i + \frac{1}{2}\left(v_i + v_f\right) t \\
x_f &= x_i + v_i t + \frac{1}{2} a t^2 \\
v_f^2 &= v_i^2 + 2a\left(x_f - x_i\right)
\end{aligned}
$$

1. $\boxed{\text{2 points}}$ An automobile is moving down a straight road and brakes with a constant acceleration (remember that acceleration in physics can be positive [speeding up] or negative [slowing down] but we use the same term–acceleration). The initial velocity is $v_i = 35$ and it slows to $v_f = 15$ in 7.0seconds,

   (a) what was the average acceleration?

   Solution:

   $$\bar{a} = \frac{15\frac{\text{m}}{\text{s}} - 35\frac{\text{m}}{\text{s}}}{7.0\text{s}} = 0.85\frac{\text{m}}{\text{s}^2}$$

   (b) How far did it travel?

   Solution: Let $x_i = 0$ then,

   $$x_f = x_i + \frac{1}{2}\left(v_i + v_f\right) t$$

   $$x_f = 0 + \frac{1}{2}\left(35\frac{\text{m}}{\text{s}} + 15\frac{\text{m}}{\text{s}}\right) 7.0 \text{ s}$$
   $$x_f = 0.85 \text{ m}$$

   or if we are told that the acceleration is constant,

   $$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

   $$x_f = 0 + (35 \times 7.0) + \frac{1}{2}\left(0.85 \times 7.0^2\right) = 0.85 \text{ m}$$

2. $\boxed{\text{4 points}}$ An small airplane has to reach a speed of $27.8\frac{\text{m}}{\text{s}}$ to takeoff. It can accelerate at $2.00\frac{\text{m}}{\text{s}^2}$. What is the minimal length of runway that would allow for a safe takeoff?

Solution:

We are given the final speed, v_f since we are told that the minimal speed for takeoff is $27.8\frac{\text{m}}{\text{s}}$. We can assume that the initial speed is $v_i = 0$ since most airplanes start their takeoff at rest waiting for clearance. We also know that the airplane can accelerate at $2.00\frac{\text{m}}{\text{s}^2}$. What we are being asked to find is the runway length, $\Delta x = x_f - x_i$. What equation on our list has all of those variables?

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = v_i^2 + 2a\left(\Delta x\right)$$

Rearrange that equation looking for Δx:

$$\frac{v_f^2 - v_i^2}{2a} = \Delta x$$

and plug in our values:

$$\frac{\left(27.8\frac{\text{m}}{\text{s}}\right)^2 - 0}{2\left(2.00\frac{\text{m}}{\text{s}^2}\right)} = \Delta x = 0.85 \text{ m}$$

3. $\boxed{\text{4 points}}$ You throw a ball up into the air. How fast should you throw it up to ensure that it will return to your hand in 2 seconds?

Solution:

As discussed in class, it takes the same amount of time for something to go up as it takes for it to go down. And so, we can simplify our work by finding the time it takes to reach the maximum height, and then multiply by 2 to get the time it takes to return to our hand.

Assume $v_f = 0$ since the velocity goes to zero at the maximum height. Assume $t = 1$ seconds since the entire trip takes 2 seconds, and 1 second is how long it takes to reach maximum height. We don't know the height, but we don't need it if we use the equation:

$$v_f = v_i + at$$

Where of course we know that $a = 9.8\frac{\text{m}}{\text{s}^2}$, so that:

$$0 = v_i + (-9.8)\frac{\text{m}}{\text{s}^2} \times 1.0 \text{ s}$$

Algebraically solve for v_i:

$$v_i = (-9.8)\frac{\text{m}}{\text{s}^2} \times 1.0 \text{ s} = 0.85\frac{\text{m}}{\text{s}}$$

4. $\boxed{\text{8 points}}$ There is an evil pig 200 meters from you. You wish to hit the pig with a cannon ball. Assume the cannon ball is launched at $y = 0$ and the pig is also at $y = 0$ on the ground. Find a combination of initial launch velocity and angle that will enable you to hit the pig. *Hint: you can pick any angle, but pick a reasonable angle.* **Bonus: What angle would allow you to hit the pig faster than any other angle and why?**

Solution: This is the two dimensional problem, where we now have to think about both the horizontal and vertical directions. First let's account for what we do know:

$$\begin{aligned} x_i &= 0 \\ x_f &= 200 \text{ m} \\ y_i &= 0 \text{ m} \\ y_f &= 0 \text{ m} \end{aligned}$$

We can pick any reasonable angle, so lets pick 50°. This means that $v_y = v\sin(50)$ and $v_x = v\cos(5)$.

First we find out limiting time from the y-component, by looking at how long it takes to reach maximum height:

$$v_{yf} = v_{yi} - 9.8t$$

Since we know that at maximum height: $v_{yf} = 0$, then,

$$0 = v_{yi} - 9.8t = v\sin(50) - 9.8t$$

$$v = \frac{9.8t}{\sin(50)}$$

That equation has two unknowns, so we need another equation. We know that it has to travel exactly 200 meters in the x direction to hit the pig, so we can use the equation (substituting 2t for t since it takes twice as long for the cannon to land as it does to reach maximum height):

$$x_f = x_i + v_i(2t) + \frac{1}{2}a(2t)^2$$

and since there is no horizontal acceleration, then we get

$$x_f = x_i + v_i(2t)$$

plugging in what we know,

$$200 = v\cos(50)(2t)$$

where we have the initial position $x_i = 0$ and substitute $2t$ instead of t since this equation is looking at the total trip, not the half-way point of max-height.

Now we have two equations with two unknowns, and can solve.

$$200 = \left(\frac{9.8t}{\sin(50)}\right)\cos(50)2t$$

We rearrange and solve for t to get $t = 3.49$ seconds. That means that it takes $2t = 6.98$ seconds for the cannon to hit the pig, but only $t = 3.49$ seconds for it to reach maximum height.

Plug that back into the equation we had for velocity:

$$v = \frac{9.8t}{\sin(50)} = 44.61\frac{\mathrm{m}}{\mathrm{s}}$$

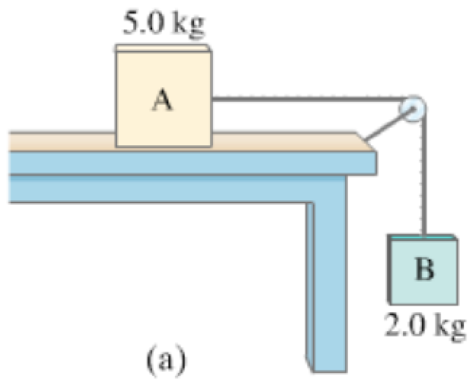
And to confirm, test it:

$$x_f = x_i + v_i(2t) = 0 + 44.61\cos(50)(2 \times 3.49) = 200\ \mathrm{m}$$

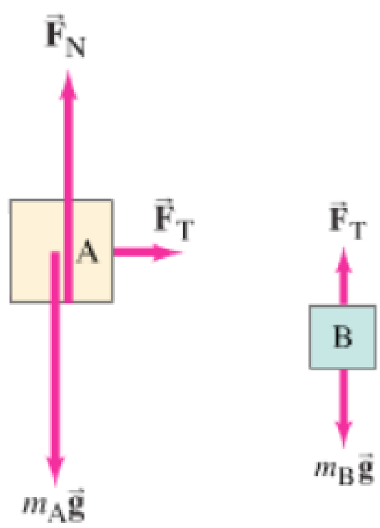
As for the bonus, this was a bit ill-posed since obviously the smaller the angle you pick, the faster you have to shoot the cannon ball, and so on.

End of exam

5. $\boxed{\text{8 points}}$ In the figure below, two boxes are attached via rope and pulley. One box is on a *frictionless* table and the other is hanging from the rope.



(a) Draw the free body diagram for each block.



Solution:

(b) Write the Newton's 2nd Law equation ($\sum F = ma$) for both boxes.

Solution: For Box B, the equation is simple and in one dimension:

$$\sum F_y = \mathbf{F}_T - \mathbf{F}_g = -m_B a$$

where we have thrown in a minus sign to indicate that the acceleration is downwards. For Box A, we have both the x and y direction to worry about, so

$$\sum F_x = \mathbf{F}_t = m_A a$$

and

$$\sum F_y = \mathbf{F}_N - \mathbf{F}_{\mathbf{g}} = 0$$

The latter equation is of no help, the first two are.

(c) Find the acceleration of the boxes.

Solution: The acceleration of the boxes will be of the same magnitude but of different directions. For Box B, acceleration will be downward and negative, for box A, acceleration will be rightward and thus positive.

From Box A:

$$\mathbf{F}_t = m_A a$$

and plugging this into the equation for Box B gives:

$$m_A a - m_B g = -m_B a \quad \rightarrow \quad -m_B g = -a(m_A + m_B) \;\rightarrow\; a = \frac{m_B g}{m_A + m_B} = 0.85 \frac{\mathrm{m}}{\mathrm{s}^2}$$

downward for Box B, and rightward for Box A.

(d) **Bonus:** If box B is 1 meter off the ground, how long will it take for it to hit the ground?

Solution: The best equation for this is:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

where $x_i = 1$ meter, $x_f = 0$, and we should assume (because it isn't told to us otherwise) that $v_i = 0$, so that,

$$0 = 1.0 + 0 + \frac{1}{2}\left(-0.85\right) t^2$$

We can solve for t, to find,

$$t = 0.85 \text{ s}$$