Physics 185 Week 5 Lecture Notes

October 28, 2012

1 Newton's First Law

Newton proposed three "Axioms, or Laws of Motion". The first is the most basic, and has been paraphrased in some textbooks as simply saying that *When a body is left alone, it maintains a constant velocity.* As Newton put it:

> Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.
>
> *Issac Newton*

This law is perhaps partially attributable to Galileo Galilei who used the term **inertia** as a name for the tendency of objects to resist a change in their state of motion, or lack thereof. Newton improved upon Galileo's work by associating **inertia** by associating the change in a state of motion with the concept of net forces acting on the body. Newton mentions, for instance, that projectiles maintain a constant velocity in their horizontal directions if we don't consider "resistance of the air", but fall towards the Earth due to gravity.

Inertial Reference Frame

Imagine you are driving down the I-76 and you take an exit ramp. You are going around a corner a bit faster than you should, and you have something sitting on your car's dashboard. As you take the curved turn, it starts to slide. The state of motion of that object is changing even though it seems there are no forces acting on it. So what is going on?

It turns out that your *frame of reference*, which is to say, how you view the world and the velocities and trajectories and forces around you, is not an **Inertial Frame**. An Inertial Frame is a postulated sort of universal frame of reference, or point of view, in which all objects only change their motion when forces are acting upon them. Put a bit simplistically, it is a frame which itself is under no state of acceleration relative to the rest of the universe.

For the problems we deal with in this chapter, we can set the Earth as being our Inertial Frame, even though, realistically the Earth is itself accelerating relative to the sun, local stars, the galaxy, and so on. We can ignore this fact for now.

Imagine you drove a friend to the train station and you can see them through the window as the train accelerates forward. They have a seat with a table, and as the train accelerates, their cell-phone which was originally resting on the table slides backwards. According to your friend's perspective, the cell phone has seemingly accelerated backwards without a corresponding net force. According to you, the cell phone's inertia caused it to slide backwards as it resisted the acceleration of the train because the table didn't have enough friction to compel it forward. You, standing on Earth, would be in an Inertial Frame, while your friend who is **on an accelerating frame, is on a non-inertial frame where Newton's laws don't apply.**

← Real world

2 Newton's Second Law

2.1 Statement of ...

Newton was rarely straight-forward, and states the second law as follows:

> "The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."
>
> *Issac Newton*

It can be stated more clearly that the response of a body to a force, in an inertial frame, is a change in the state of motion. By change of the state of motion we mean that the velocity changes in either magnitude or direction, or both. We call such a change *acceleration*.

Stated even more clearly, the acceleration of a body is proportional to the force being exerted on it. Even more precisely, the acceleration is proportional to the **net force**, the sum of all forces acting on the body, and along the same direction as the net force.

As precisely as we can get is the following statement of Newton's second law:

In an inertial frame, a net force acting on a body results in an acceleration of that body which is equal to the net force divided by the mass of the body, or simply put: Force equals mass times acceleration.

Newton's Second Law

$$F_{\text{net}} = ma$$

2.2 Examples of ...

Let's consider two similar examples with very different outcomes. A ball is resting on a table. What are the net forces acting on the ball? The gravitational force pulls the ball downward, and the table is providing a *support* or *normal* force in the opposite direction holding the ball up. The net force acting on the ball is zero since the *normal* force upwards is balancing out the gravitational force downwards.

Now let's suppose that a sudden draft blows the ball off the table (which of course is a force unto itself). What are the forces acting on the ball as it falls to the ground? There is only one such force– the gravitational force downward. *Whenever there is only one force acting on an object, the object can not have a net force of zero.*

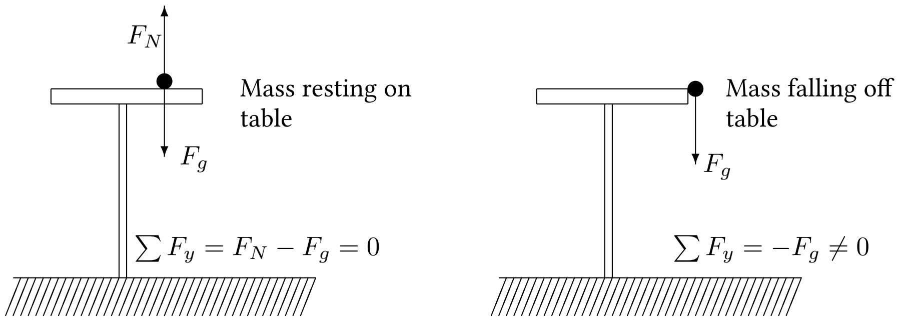


Figure 2.1: **Left:** The ball is resting on the table such that the normal force of the table holding the ball up cancels out the gravitational pull on the ball downwards, and so we say that the net forces $\sum F = F_N - F_g = 0 \quad \rightarrow \quad F_N = F_g$. **Right:** The ball is in free-fall and has only one force acting on it, thus the ball has a net force of $\sum F = F_g = ma$. The acceleration due to gravity on Earth is $g = -9.8\frac{\text{m}}{\text{s}^2}$. Drawing the forces as vectors on the object is called a **Force diagram** or **Free body diagram**.

Mechanical Equilibrium

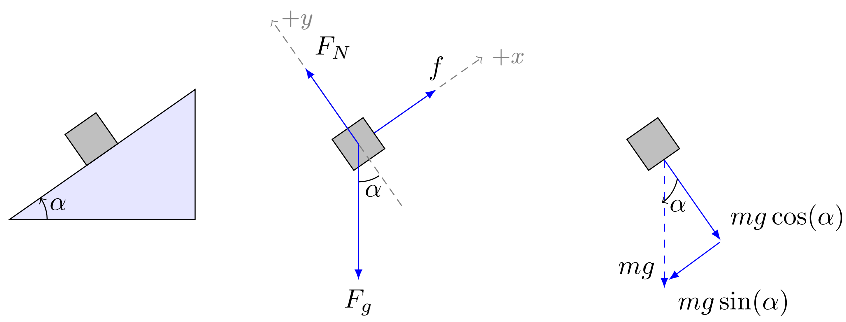
An object is in mechanical equilibrium if the sum of all the forces acting on the object is zero, that is, if all the forces acting on an object cancel out, or if there are no forces acting on the body. $\sum F = 0$.

2.3 Quantitative Solutions

Example One: Consider a block sitting on an inclined plane that has an angle of $30°$. There is enough friction between the block and the inclined plane that the block will not slide down. The mass of the block is 3 kilograms. What is the force of friction and the normal force? *Given:* $g = 9.8\ \frac{\text{m}}{\text{s}^2}$.

Steps to solve 2nd Law Problems:

- Draw a rough sketch of the problem.
- Label the sketch with forces. Every force should get a corresponding vector.
- Choose how to coordinate your axis system. It is generally easiest to line the x, y axis so that most of the forces are parallel with the x or y axis.
- For those forces which aren't parallel to the x or y axis, we have to decompose them into vectors which are parallel to the x and y axis such that the sum of the decomposed vectors adds up to the original vector. **It will always be easier to do this by drawing a second picture which zooms in on this vector.**
- Now you can find the sum of all vectors in the x direction and the y direction. By Newton's laws, the sums $\sum F_x$ and $\sum F_y$ should add to zero if the system is under equilibrium. If the sum of the vectors don't add to zero, then there will be a net acceleration in that direction.



In the sketches above at left, we draw a simple outline of the ramp and weight structure. The angle the ramp makes with the floor is α. In the middle sketch, we focus in on the weight and label the forces. We know where to put them as follows:

How to know where to put the forces:

- Normal forces are forces that hold up masses, for example, when you stand on the floor you are pushing down with your weight, and the floor is pushing back. This pushing back is called a "Normal Force" and is labeled F_N. **A Normal Force always points in a direction perpendicular to the surface of contact.** Thus the normal force in this case should be perpendicular to the ramp.
- Friction forces always point in the opposite direction of motion, *or when there is no motion, in the opposite direction of the motion that would occur were there no friction.* In this case, if there were no friction, the block would slide down the ramp, so the friction points up the ramp. We label this force f.
- Gravity always points towards the center of the Earth and is labeled F_g.

We have chose our xy axis so that the x axis is parallel to the ramp and points up the ramp (and thus parallel to the friction force). The y axis is automatically perpendicular to this axis, but we have a choice to have it pointing upwards or downwards, and we choose to point it upwards so that it is parallel with the Normal Force.

But we have a problem: The gravitational force is not parallel to either of our axis. We have to break this up into components in the x and y direction. By some geometry (similar triangles or compliments of a right angle), we find the angle between F_g and the y-axis is α as well, and so we find:

$$F_{g,x} = mg\sin(\alpha), \quad F_{g,y} = mg\cos(\alpha)$$

Now that we have all of our forces into their x and y axis components, we can apply Newton's Second Law:

$$\begin{aligned} \sum F_x &= f - F_g\sin(\alpha) = 0 \\ \sum F_y &= F_N - F_g\cos(\alpha) = 0 \end{aligned}$$

We knew that both of those net force equations were equal to zero because the block is in *equilibrium*, that is, there is no change to its state of motion. We can thus solve for the friction force and the normal force:

$$f - mg\sin(\alpha) = 0 \;\rightarrow$$
$$f = mg = 3.0\,\text{kg}\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)\sin(30) = 14.7\,\text{N}$$

Where we know that this is pointing up the ramp based on our sketch, and we recall that the Newton is a derived SI unit:

$$\text{kg}\frac{\text{m}}{\text{s}^2} = \text{N}$$

As for the normal force:

$$F_N - mg\cos(\alpha) = 0 \;\rightarrow$$
$$F_N = mg\cos(\alpha) = 3.0\,\text{kg}\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)\cos(30) = 25.5\,\text{N}$$

As a check, we can find the resultant force of these two forces, where we find that

$$\sqrt{14.7^2 + 25.5^2} = 29.4\,\text{N} = F_g$$

Example One (b): What is the coefficient of static friction from the previous example?

Static and Kinetic Friction

It has been experimentally found that the force of friction can be closely approximated as being proportional to the normal force between the object experiencing friction and the surface on which it is experiencing friction. That is,

$$f = \mu F_N$$

where μ is a constant that depends on the types of materials involved. But in fact we have two different types of friction, **static** and **dynamical**. Recall from a previous lecture that we had dynamical mechanical equilibrium (where the net forces were zero but the object had a constant straight-line motion) and static mechanical equilibrium (where the net forces were zero and there was no velocity.

Static and Kinetic Friction

Static friction is written $f_s = \mu_s F_N$ and is the friction experienced by an object which is being pushed or pulled against a surface, but which is not in motion because the friction forces are too strong. Try pushing your hands very hard against each other, so hard that when you try to slide them, they don't move. That is static friction. Eventually if you push hard enough, they will suddenly slide against each other–then they will be experiencing **Kinetic friction**, which is written $f_k = \mu_k F_N$. Kinetic friction is weaker than static friction, which is why when your hands suddenly slide against each other, it is a very rapid and sudden motion–this is the basics behind why earthquakes are so deadly.

It will always be the case that for every material:

$$\mu_k < \mu_s$$

In the previous problem, we found the $f = 14.7$ N and $F_N = 25.5$ N, and so we have

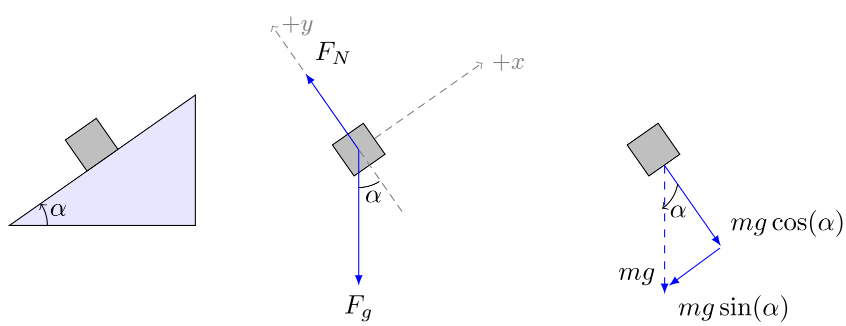
$$f = \mu_s F_N$$

$$14.7 \text{ N} = \mu_s 25.5 \text{ N}$$

$$\mu_s = 0.576$$

If the block had been in motion, these would have been the numbers for μ_k.

Example Two: In the previous problem, suppose there is now no friction. What is the net acceleration of the block?



In this case, the sum of the forces is:

$$\sum F_x = -mg\sin(\alpha) = ?$$

$$\sum F_y = F_N - mg\cos(\alpha) = 0$$

We know that the forces in the y direction will balance out since we assume that ramp isn't collapsing, but now that there is no friction, we physically know that the block will slide down the ramp. Thus we apply Newton's second law:

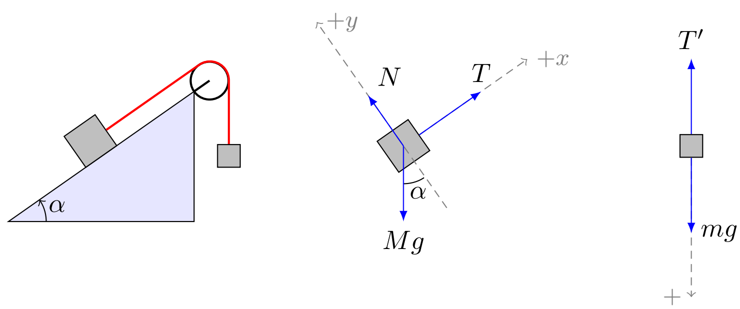
$$\sum F_x = -mg\sin(\alpha) = ma$$

We can divide m out of both sides and find that the acceleration is

$$a = -g\sin(\alpha) = -4.9\frac{\text{m}}{\text{s}^2}$$

Which confirms our physical intuition (not to mention common sense) that the block will slide down the ramp. But note that it isn't accelerating as fast as the gravitational acceleration.

Example 3: Suppose we have a weight with mass $M = 5$ kg that is on a ramp. It is connected to a mass of weight $m = 2$ kg by a rope and pulley system. What is the acceleration, and in what direction, of the two masses?



We can treat the two masses as two separate problems. The simplest scenario is the hanging mass, where all of the forces are acting in a single direction:

$$\sum_m F_y = T' - mg = ma$$

We don't know what the tension will be, yet, but we do know that tension in a rope is uniform throughout the rope so that $T = T'$. Also, since the masses are tied together, they share the same acceleration, a.

For the larger block, we can do the same decomposition of the gravitational force into vectors parallel and perpendicular to the slope as we did in the previous problems. Doing so yields the following two second law equations:

$$\sum_M F_x = T - Mg\sin(\alpha) = Ma$$

$$\sum_M F_y = F_N - Mg\cos(\alpha) = 0$$

Notice that the sum of the forces along the ramp for this mass results in Ma not ma, since the mass of this block is different. Also notice that the sum of the forces perpendicular to the ramp are zero, as they should be. We thus have the three equations:

$$\begin{aligned} T - Mg\sin(\alpha) &= Ma \\ T - mg &= ma \end{aligned}$$
$$F_N - M_g\cos(\alpha) = 0$$

This is wrong!

To see why this is wrong, we have to remember that the two masses are tied together, so that they move in the same direction. Thus they should have *the same sign* for that direction. If we set going up the ramp as positive for mass M, then we need to also set going downwards as positive for the mass m. This *has* to be done to ensure that the acceleration can be solved properly.

Thus if we set down as positive for the mass m, then the net force equation should be

$$-T + mg = ma$$

The two equations we are concerned with are now:

$$\begin{aligned} -T + mg &= ma \\ T - Mg\sin(\alpha) &= Ma \end{aligned}$$

We can add these two equations together to get:

$$(m - M\sin(\alpha))g = (m + M)a$$

Then we have

$$a = \frac{m - M\sin(\alpha)}{m + M}g = -0.7\frac{\mathrm{m}}{\mathrm{s}^2}$$

This tells us that the acceleration will be in the direction of down the ramp for mass M, since the sign is negative, and upwards for mass m. This sort of makes sense if we consider that $M > m$, but this wouldn't necessarily be the case for a smaller angle. In fact, the acceleration would be zero for the angle at which $m - M\sin(\alpha) = 0$. We can solve for this by finding,

$$\alpha = \sin^{-1}\left(\frac{m}{M}\right) = 23.6°$$

Or we could keep the same angle but change one of the masses, say m. The two blocks would balance out at $a = 0$, so that

$$m' - 5\text{ kg}\sin(30°) = 0 \quad \rightarrow \quad m' = 2.5\text{ kg}$$

3 Newton's Third Law

> To every action there is always opposed an equal reaction.
>
> *Issac Newton*

In a sense we have already been talking about Newton's third law when we were discussing normal forces, such that of a table holding a block or a ball up. The table is exerting a reacting force against the downward force of the weight of the ball. This is an equal and opposite force. When you push against a wall, the wall is pushing back with equal force.

Even when a block is on the ramp, the Normal force is an equal and opposite force. It is only that in such cases the component of gravity directed perpendicular to the surface of the ramp is not as great as it would be if the block were simply resting on a flat surface.

You are yourself exerting an equal an opposite force on the Earth. While the Earth is pulling you downwards, you are pulling it upwards. But because the Earth is so unimaginably more massive than you are, we can correctly ignore your pull on the Earth, but you can never ignore the Earth's pull on you.

Newton's third law has some deeper implications that we'll get to in the next lectures.