PHYS 305 - Assignment #3

Make sure your name is listed as a comment at the beginning of all your work.

Purpose: Illustrate error calculations in solving ODEs

Mass on Spring

Newton equation for the mass on the spring problem is particularly simple to solve analytically and numerically as well. This makes it ideal to study the accuracy of ODEs solver algorithms by a direct comparison between the approximate solution and the exact solution.

Another way to check the accuracy of the approximate solution is to use *conserved quantities* that result from symmetries in the ODEs. For instance, the total energy for the mass on spring problem must be conserved, i.e., it must remain a constant as a function of time.

Part A (5pts)

Define the mass on the spring problem as in the web pages on *Newton's equation of motion: simple scheme* ($k = 4.0$ and $m = 1.0$), but use the initial conditions $x = 0.5$ and $v = 1.7$.

Write a C/C++ code implementing the *Euler method* (in which a is constant over each Δt step) as as a starting point in developing a new code.

Run the code for several different values of Δt—0.016, 0.008, 0.004, 0.002, 0.001, 0.0005, 0.00025, and 0.000125 (i.e. decreasing by factors of 2)—and investigate how the errors depend on the time step Δt. Specifically, rather than choosing a specific time at which to evaluate the difference between the numerical ("num") and analytical ("an") solutions, define the *maximum errors* by

$$\begin{aligned}
\Delta x &= \max_{1\le i\le n} \left| x_{num}(t_i) - x_{an} \right|, \\
\Delta v &= \max_{1\le i\le n} \left| v_{num}(t_i) - v_{an} \right|, \\
\Delta E &= \max_{1\le i\le n} \left| \tfrac{1}{2} m\, v_{num}(t_i)^2 + \tfrac{1}{2} k\, x_{num}(t_i)^2 - E \right|,
\end{aligned}$$

where k and m are the spring constant and the mass respectively, n is the total number of points and E is the (conserved) energy of the system (figure it out!). Construct a table of Δx, Δv, and ΔE versus Δt for the values of Δt listed above. Using a power law fit of Δ versus Δt, determine the exponent α in the relations $\Delta \propto \Delta t^{\alpha}$. What is the advantage of using E instead of x or v to check the accuracy of the solution?

Part B (5pts)

Repeat the above section using the *Midpoint Method.* As a comment in your shell script describe how your results show, for this system, that the Midpoint method is more accurate.