# PHYS 305 - Assignment #6

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*Make sure your name is listed as a comment at the beginning of all your work.*

*Purpose:* Develop a physical intuition for the organization of a chaotic solution.

## Lotka-Volterra Model by hand

You will solve this section ysing paper and pencil and turn it in during next weeks rec. Consider the modified Lotka-Volterra model where $x > 0, y > 0$:

$$\dot{x} = x(3 - x - 2y) \tag{1}$$

$$\dot{y} = y(2 - y - x) \tag{2}$$

Find all 4 fixed points by hand. Compute the Jacobian matrix for each fixed point and classify each one (stable, unstable, saddle). Sketch the trajectory near each fixed point. Shade in the basin of attraction for the fixed point on the positive part of the x-axis.

## The Lorenz Butterfly Attractor

Lorenz used the parameter values $\sigma = 10.0$, $B = 8.0/3.0$ and $R = 28.0$ in his original studies of the *Lorenz Butterfly Strange Attractor*. We will now look in greater details at this exotic attractor. Use the code from the web to generate the data for this assignment. Eliminate transients by running the code for t=3 and take data up to t=15.

- Fixed Points
  - Overlay a *symbol* (i.e., a small circle, square, . . . ) on the 3-D graph of the attractor to mark the positions of the fixed points
  - Do the same for the projected graphs, i.e., $z$ vs $x$, $y$ vs $x$, ...
- Max/Min Map of the Lorentz Attractor

  Lorenz in his original seminal work observed that *a trajectory leaves one spiral only after exceeding some critical distance from its center*. Moreover, the extent to which this distance is exceeded appears to determine the point at which the next spiral is entered; this in term seems to determine the number of circuits to be executed before changing spirals again. This implies that a maximum of z suffices to predict the next maximum of z.

- Write an analysis code that finds the all of the maxima and/or minima of $z(t)$. The code should decide what to calculate based on line arguments that could be $\langle -\text{max} \rangle$ and/or $\langle -\text{min} \rangle$. The code should then read the result of solving the Lorenz model from $\langle \text{stdin} \rangle$. It should detect the maxima (minima) in $z(t)$ and then calculate the exact value of each maximum (minima) via a three points parabola interpolation based on function values at three adjacent points on the numerical lattice $dt$ apart.
- Plot adjacent maxima of $z(t)$ versus each other, i.e., $z^{k+1}_{max}$ versus $z^{k}_{max}$ for all $k$.
- Repeat the plot above, this time plotting minima versus adjacent minima.

• Poincare Surface of Section

A *Poincare Surface of Section* records the crossing of a trajectory with an arbitrarily chosen plane. This builds a 2-dimensional *map* based on the location of the sequential crossings in the plane.

- Plot $z(t)$ versus $t$ from the *Lorenz Attractor* over some small time domain. Note the complexity of this graph.
- Write a program to calculate the *Poincare Section* of the *Lorenz Attractor*. Define this *Poincare Surface of Section* as the intersection of the trajectory with the plane $z = 37.0$. The code should find when the trajectory crosses the plane and then use linear interpolation to define precisely the location of the intersections with the plane.
- Produce two images: one that captures all crossings with the plane and a second that records only to the crossings from below to above the plane.

The *Lorenz Butterfly Attractor* is a *fractal object* and therefore chaotic. Yet, there is order in this chaotic solution!