PHYS 305 - Assignment #8

Due: Friday, March 6^{th}

Make sure your name is listed as a comment at the beginning of all your work.

Purpose: Develop a physical intuition for unstable periodic orbits (embeded in a chaotic solution) and understand the lattice refinement technique proposed by Henry, Watt and Wearne to find periodic orbits..

Periodic Orbits

Henry, Watt and Wearne proposed the lattice refinement technique in 2000. The technique is described in the section on *Chaotic Scattering* and coded in the code *search_for_orbits.c* in the web. This code searches for periodic orbits in the three repellor hills potential.

Part 1: Illustrate the orbits suggested by *search_for_orbits.c*.

- Understand and run the code *search_for_orbits.c*.
- Run your *scattering code* with the suggested initial conditions for orbits that exhibit very different different characteristics (i.e., orbits that *look different to your eyes*). Overlay equipotential lines of adequate radius to illustrate intuitively the likelihood of the shapes you observe for the orbits. Adjust your plot (by adjusting the x- and y-range) for the periodic orbits to occupy most of the plotting area.
- Illustrate the different degree of instability for these orbits by plotting the trajectories for integration times $2T$, $3T$, $4T$, ... where T is the period of the orbit. Modify *search_for_orbits.c* to output this period T. Comment on what you find.

Part 2: Find new periodic orbits.

- Modify the code *search_for_orbits.c* to search for orbits outside of the search window defaulted in the code.
- Modify the code *search_for_orbits.c* to search for orbits of different *topology* than in Part 1. In particular, search for orbits that have *no perpendicular crossings with the x-axis*.

Part 2: Find periodic orbits in the Lorenz butterfly chaotic attractor.

- Modify the code *search_for_orbits.c* to search for orbits in the Lorenz system This is the original problem described in the paper by Henry, Watt and Wearne (2000)
- Find the periodic orbits and plot them overlaid on the Lorenz butterfly chaotic attractor.