

# PHYS 305 - Assignment #3

Due: Friday, January 30<sup>th</sup>

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*Make sure your name is listed as a comment at the beginning of all your work.*

*Purpose:* Illustrate error calculations in solving ODEs

## Mass on Spring

Newton equation for the mass on the spring problem is particularly simple to solve analytically and numerically as well. This makes it ideal to study the accuracy of ODEs solver algorithms by a direct comparison between the approximate solution and the exact solution.

Another way to check the accuracy of the approximate solution is to use *conserved quantities* that result from symmetries in the ODEs. For instance, the total energy for the mass on spring problem must be conserved, i.e., it must remain a constant as a function of time.

### Part A

Define the mass on the spring problem as in the web pages on *Newton's equation of motion: simple scheme* ( $k = 4.0$  and  $m = 1.0$ , but use the initial conditions  $x = 0.5$  and  $v = 1.7$ ). Use the in-class code implementing the *simple Newton solution* (in which  $a$  is constant over each  $\delta t$  step) as a starting point in developing a new code.

Run the code for several different values of  $\delta t$ —0.016, 0.008, 0.004, 0.002, 0.001, 0.0005, 0.00025, and 0.000125 (i.e. decreasing by factors of 2)—and investigate how the errors depend on the time step  $\delta t$ . Specifically, rather than choosing a specific time at which to evaluate the difference between the numerical (“num”) and analytical (“an”) solutions, define the *maximum errors* by

$$\begin{aligned}\Delta x &= \max_{1 \leq i \leq n} |x_{num}(t_i) - x_{an}|, \\ \Delta v &= \max_{1 \leq i \leq n} |v_{num}(t_i) - v_{an}|, \\ \Delta E &= \max_{1 \leq i \leq n} \left| \frac{1}{2} m v_{num}(t_i)^2 + \frac{1}{2} k x_{num}(t_i)^2 - E \right|,\end{aligned}$$

where  $k$  and  $m$  are the spring constant and the mass respectively,  $n$  is the total number of points and  $E$  is the (conserved) energy of the system (figure it out!). Construct a table of  $\Delta x$ ,  $\Delta v$ , and  $\Delta E$  versus  $\delta t$  for the values of  $\delta t$  listed above. By a power law fit of  $\Delta$  versus  $\delta t$ , determine the exponent  $\alpha$  in the relations  $\Delta \propto \delta t^\alpha$ .

What is the advantage of using  $E$  instead of  $x$  or  $v$  to check the accuracy of the solution?

## Part B

Implement the solution of the mass on spring problem using the Mid-Point method. Check the accuracy of the ODE solution as done above for this solution.