PHYS 305 - Assignment #1

Due: Friday, January 16^{th}

Make sure your name is listed as a comment at the beginning of all your work. Your work must be sent as a single tar file.

Purpose: Accuracy analysis in summing series and programming practice.

Using C or C++ and gnuplot

1. Write a program to sum the series

$$\zeta(s) \equiv \sum_{n=1}^{\infty} n^{-s}$$

for $s = 2$. Stop your calculation when the next term in the series is less than $\epsilon = 10^{-6}$ times the current sum. Print out your result to 9 decimal places. How many terms in the sum are needed? Repeat with $\epsilon = 10^{-9}$. Does it make a difference if you carry out the calculations in double precision (`double`) instead of single precision (`float`)? Why or why not? This question should be answered as output to the user.

2. Repeat for $\zeta(4.5)$.

3. Write a program to sum the series

$$f_N(x) = \sum_{n=1}^{N} \frac{2}{\pi} \left[1 - (-1)^n\right] \frac{\sin(nx)}{n} .$$

This is the Fourier series representation of the step function

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi . \end{cases}$$

Plot $f(x)$, $f_1(x)$, $f_5(x)$, $f_{10}(x)$, and $f_{100}(x)$ on a single graph, for $0 \leq x < \pi$ in steps of $\delta x = \pi/500$.

4. What value of N is needed to determine the integral $\int_0^\pi \sin x \, dx$ to a relative accuracy of $\epsilon = 10^{-4}$? (How do you go about determining the relative accuracy in this case?) What value of N is needed for $\epsilon = 10^{-6}$? Answer this question using the trapezoidal rule and the Simpson rule.