PHYS 115 Rec #4

Using MAPLE to solve linear circuit equations

Now that you've learned the analytical tools to solve circuits with resistors and sources of emf, lets use the power of symbolic algebra to streamline the process. Given a circuit, we can write down independent equations which can be solved with Linear Algebra. While it is not a co-requisite for this course, as physics majors you will be taking this topic soon enough. This assignment is meant as a gentle introduction to one of the practical applications of Linear Algebra.

Sample circuit

Let us first look at a simple circuit with two resistors in parallel connected to a battery and a resistor in series. Using the loop and node rules we get the following three equations:

$$V_1 - I_1 R_1 - I_3 R_3 = 0 \tag{1}$$
$$V_1 - I_2 R_2 - I_3 R_3 = 0 \tag{2}$$
$$I_1 + I_2 - I_3 = 0 \tag{3}$$

These equations can be represented by the 'linear equation':

$$\mathbf{Mc} = \mathbf{s} \tag{4}$$

Where M is a matrix, s and c are vectors. The s vector is the RHS of the equation, all the terms that do not contain any of our unknowns (I_1, I_2, I_3). Thus we have:

$$s = \begin{bmatrix} -V1 \\ -V1 \\ 0 \end{bmatrix} \tag{5}$$

The c vector is a vector of our unknowns,

$$c = \begin{bmatrix} I1 \\ I2 \\ I3 \end{bmatrix} \tag{6}$$

and the M matrix represents the coefficients in front of each of our unknowns:

$$M = \begin{bmatrix} -R1 & 0 & -R3 \\ 0 & -R2 & -R3 \\ 1 & 1 & -1 \end{bmatrix} \tag{7}$$

The solution to such an equation can be found by taking the matrix inverse,

$$\mathbf{c} = \mathbf{M^{-1}s} \tag{8}$$

Using MAPLE

Lets see how we can use MAPLE to do all the messy algebra for us. To begin, open MAPLE from the terminal by the command:

```
> xmaple &
```

Once loaded type the following commands (this tells MAPLE to use the LinearAlgebra package, much like the include statement):

```
restart:
with(LinearAlgebra):
```

Using the example from above we can define the two vectors:

```
c := Vector( [I1, I2, I3] );
s := Vector( [-V1,-V1,0] );
```

The matrix M can be defined by the following list-of-lists:

```
M := Matrix( [ [-R1,0,-R3],[0,-R2,-R3],[1,1,-1] ] );
```

The solution can be found by the command:

```
solution := (M **(-1)) . s;
```

If you notice, this is a mess! Lets plug in some actual values, use $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $V_1 = 3/2V$. Enter the following lines at the beginning of your MAPLE code, right after the 'with' statement:

```
R1:=1;
R2:=2;
R3:=3;
V1:=3/2;
```

Now rerun the the subsequent lines and you should have your numerical solution!

$$c = \begin{bmatrix} I1 \\ I2 \\ I3 \end{bmatrix} = \begin{bmatrix} \frac{3}{11} \\ \frac{3}{22} \\ \frac{9}{22} \end{bmatrix} \tag{9}$$

Computer assignment

While the last example may seem trivial (after all, we solved it in class last week), lets move on to a circuit that would be very difficult to do by hand. Given the following circuit, with the current paths chosen for you, solve for the currents. Each resistor has its strength labeled by its number, i.e. $R_3 = 3\Omega$ and all sources of emf have a constant $5V$ potential across them. Submit your MAPLE code that prints out the solutions.