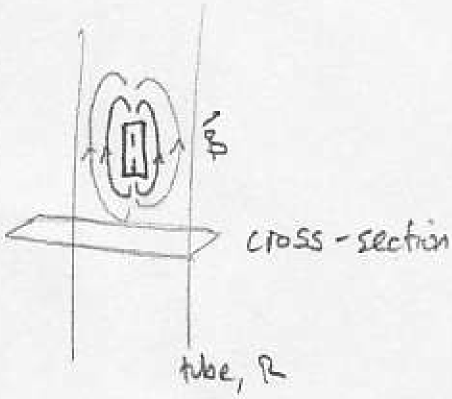
PHYS 115] HW #6 solutions

#1]

Consider a cross-section of the tube



As the magnet falls – the flux $\Phi = \oint \vec{B}\cdot d\vec{A}$ increases, this creates an emf to oppose the change in flux, This emf creates its own magnetic field, since in our cross-section we now have a current $I = \varepsilon_{ind}/R$. The $\vec{B}$ field here will slow down the magnet.

#2]

$$|\vec{B}(t)| = B_0 + bt^3 \qquad \frac{d}{dt}|\vec{B}| = 3bt^2$$

Inside $r_1 < R$

$$|\varepsilon| = \frac{d\Phi}{dt} = \frac{d}{dt}\oint \vec{B}\cdot\hat{n}\,dA = \frac{d}{dt}\pi r_1^2|\vec{B}| = 3b\pi r_1^2 t^2$$

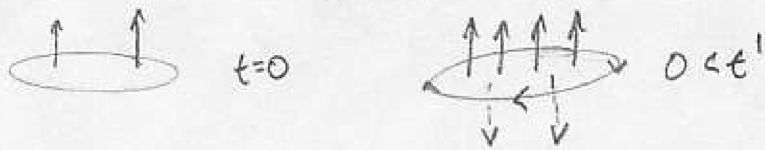
$$E = 3/2\, b r_1 t^2$$

At a larger radius $r_2 > R$ only the portion from $r \le R < r_2$ counts, hence $|\varepsilon| = 3b\pi R^2 t^2$

$$\varepsilon = \int \vec{E}\cdot d\vec{\ell} = E\, 2\pi r_2$$

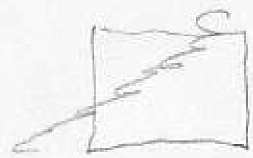
$$E = 3/2\, bR^2t^2/r_2$$

Since the flux is growing out of the page, the emf must go clockwise



#3]

$$B = .07 + .03t^2$$



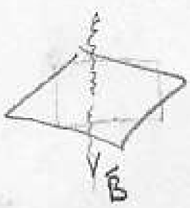
$$\frac{dB}{dt} = .06t$$

$$A_{sol} = \pi r^2 \; ; \; r = .03m$$

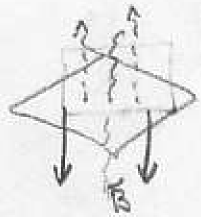
$$|\mathcal{E}_{ind}| = \frac{d\Phi}{dt} = \oint \vec{B}\cdot\hat{n}\, dA = \frac{d}{dt} N \Big[\underbrace{\int \vec{B}\cdot\hat{n}\, dA}_{\text{inside solenoid}} + \underbrace{\cancel{\int \vec{B}\cdot\hat{n}\, dA}}_{\text{outside solenoid}} \Big]$$

$$= N \cdot \frac{d}{dt}\left[B\pi r^2 \right] = N\pi r^2 (.06)t$$

$$I(t=2) = \frac{\mathcal{E}(t=2)}{R} = .0136A$$



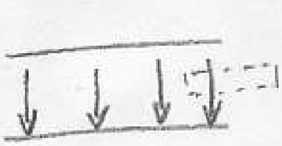
$t=0$



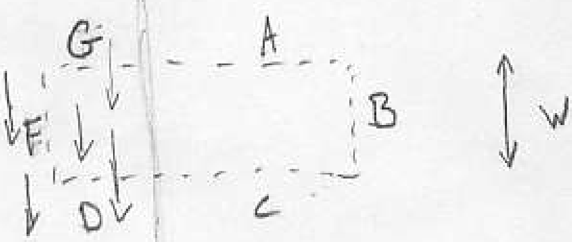
$t' > 0$

current must run clockwise in the picture shown

#4]



take path as shown

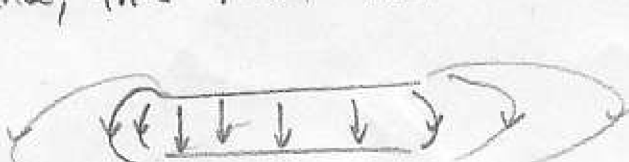


$\int \vec{E}\cdot d\vec{l} = 0$ for A, B, C since $\vec{E} = 0$

$\int \vec{E}\cdot d\vec{l} = 0$ for G, D since $\vec{E}\cdot d\vec{l} = 0$

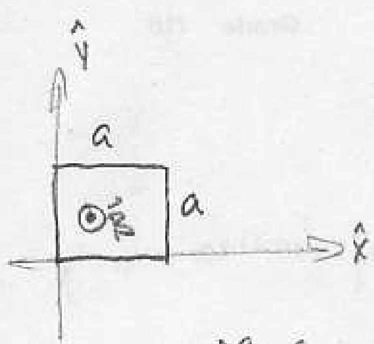
$\int \vec{E}\cdot d\vec{l} = Ew$ for path F but $\oint \vec{E}\cdot d\vec{l} = 0 \neq Ew$!

Hence, this must not be the field.



A more realistic field — fringes

#5]



$$\frac{dB}{dt} = \frac{d}{dt}\,4t^2 y = 8ty$$

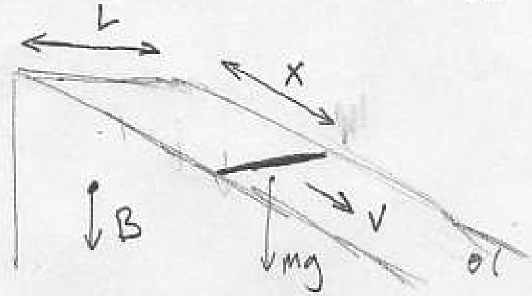
$$\Phi = \int \vec{B}\cdot\hat{n}\,dA$$

$$= \int_0^a\int_0^a \vec{B}(x,y)\cdot\hat{n}\,dx\,dy = \int_0^a\int_0^a 4t^2 y\,dx\,dy$$

$$= (4t^2 a)\left.\frac{y^2}{2}\right|_0^a = 2t^2a^3$$

Hence $|\mathcal{E}| = \frac{d\Phi}{dt} = 4t^2a^3$ $\quad \mathcal{E}(t=2s) = 8\cdot 10^{-5}\,V$, clockwise

#6]



If v is in the direction down the plane then

$$F_g = mg\sin\theta$$

$$\Phi = \int \vec{B}\cdot\hat{n}\,dA = (\vec{B}\cdot\hat{n})\,Lx = B\cos\theta\,Lx$$

$$\frac{d\Phi}{dt} = B\cos\theta\,L\,\frac{d}{dt}x = BLv\cos\theta$$

$$I_{ind} = \frac{d\Phi}{dt}\Big/R = \frac{BLv\cos\theta}{R}$$

The force on a current in a B field is $F_B = I\,\vec{L}\times\vec{B} = ILB$

but this force is not along the direction of v, $F_{B,\text{along } v} = ILB\cos\theta$

Steady state implies $F_g = F_B$, $\frac{B^2L^2\cos^2\theta\, v}{R} = mg\sin\theta$ $\quad v = \frac{mg\sin\theta}{L^2B^2\cos\theta^2}R$

rate of potential energy lost

$$= \frac{d}{dt}U = \frac{d}{dt}mg\sin\theta\,x = mg\sin\theta\,v = mg\sin\theta\left(\frac{mg\sin\theta}{L^2B^2\cos\theta}R\right)$$

same!

$$P = \mathcal{E}^2/R = \frac{(d\Phi/dt)^2}{R} = \frac{(BL\cos\theta)^2 v^2}{R} = \frac{(BL\cos\theta)^2}{R}R^2\frac{(mg\sin\theta)^2}{(L^2B^2\cos\theta)^2} = \frac{(mg\sin\theta)^2R}{L^2B^2\cos\theta}$$

*) effect would still hold if $\vec{B}\to-\vec{B}$ as the flux would be the same