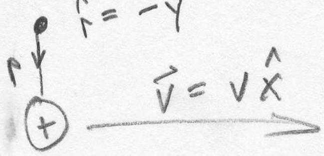
PHYS 115 HW #4

review: 41]



Lab frame: $\vec{E}_L = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2} \hat{y}$

$$\vec{B}_L = \frac{\mu_0}{4\pi} \frac{e}{r^2} \vec{v} \times \hat{r}$$

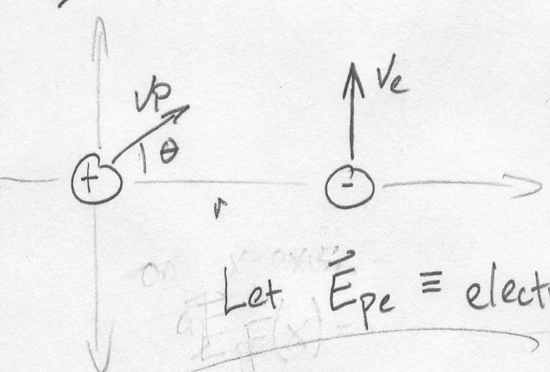
$$\vec{v} \times \hat{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -\hat{z}$$

$$= -\frac{\mu_0}{4\pi} \frac{e}{r^2} \hat{z}$$

Moving Frame: $\vec{v} = 0$ hence $\vec{v} \times \hat{r} = 0 \rightarrow \vec{B}_m = 0$

r and $\hat{r}$ are unchanged hence $\vec{E}_m = \vec{E}_L$

44] assume $v_p, v_e \ll c$, choose coordinate orgin at proton:



$$\vec{v}_p = v_p (\cos\theta \hat{x} + \sin\theta \hat{y})$$
$$\vec{v}_e = v_e \hat{y}$$
$$\vec{r}_p = 0\hat{x} + 0\hat{y} \qquad \vec{r}_e = r\hat{x}$$

Let $\vec{E}_{pe} \equiv$ electric field proton sees due to electron, then

$$\vec{E}_{pe} = \frac{1}{4\pi\epsilon_0} \frac{-e}{r^2} \hat{x} \qquad \vec{E}_{ep} = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2} (-\hat{x})$$

$$\vec{v}_e \times \hat{r}_{pe} = (v_e \hat{y}) \times (\hat{x}) = -v_e \hat{z}$$

$$\vec{v}_p \times \hat{r}_{ep} = \left(v_p(\cos\theta \hat{x} + \sin\theta \hat{y})\right) \times (-\hat{x})$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_p\cos\theta & v_p\sin\theta & 0 \\ -1 & 0 & 0 \end{vmatrix} = v_p \sin\theta \hat{z}$$

44] cont.

$$\vec{B}_{pe} = \frac{\mu_0}{4\pi} \frac{-e}{r^2} \left(- v_e \hat{z} \right)$$

$$\vec{B}_{ep} = \frac{\mu_0}{4\pi} \frac{e}{r^2} \left(-v_p \sin\theta \, \hat{z} \right)$$

$$\vec{F}_{pe} = q_e \left(\vec{E}_{pe} + \vec{v}_p \times \vec{B}_{pe} \right)$$

$$\vec{v}_p \times \vec{B}_{pe} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_p\cos\theta & v_p\sin\theta & 0 \\ 0 & 0 & \frac{\mu_0 e v_e}{4\pi r^2} \end{vmatrix} = \frac{v_p v_e \sin\theta \, \mu_0 e}{4\pi r^2} \hat{x} + - \frac{v_p v_e \cos\theta \, \mu_0 e}{4\pi r^2} \hat{y}$$

$$= \frac{v_p v_e \mu_0 e}{4\pi r^2} \left(\sin\theta \, \hat{x} - \cos\theta \, \hat{y} \right)$$

$$\vec{v}_e \times \vec{B}_{ep} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & v_e & 0 \\ 0 & 0 & -\frac{\mu_0 e}{4\pi r^2} v_p \sin\theta \end{vmatrix} = -\frac{v_e v_p \sin\theta \, \mu_0 e}{4\pi r^2} \hat{x}$$

Since $|\vec{E}_{ep}| = |\vec{E}_{pe}|$ and they are in opp. directions, we can say that the electric fields are conservative. This is not true for the $\vec{B}$ fields (look at the eq.) As for $\vec{p}$ (momentum) see the comment in 20.P43)

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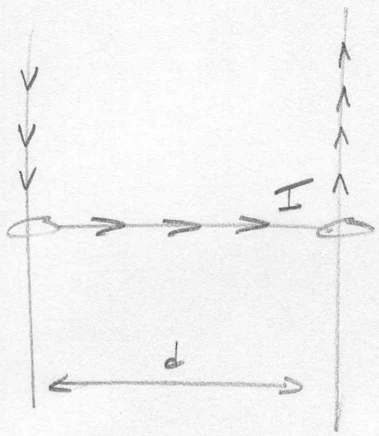

Neglect all $\vec{B}_{wire}$ (problem explictly states this)

the total number of electrons along the rod

$$N = n A d$$

where n is the number density

A is cross-section of wire

Going at speed: (to the left, they are electrons)

$$I = |q| n A \bar{v} \rightarrow \bar{v} = \frac{I}{e}\frac{1}{nA}\hat{x}$$

Thus the total force on the wire is

$$\vec{F} = N(-e)\left(\cancel{\vec{E}}^{\to 0} + \bar{v} \times \vec{B}\right)$$

$$= -\cancel{nA}d\cancel{e}\frac{I}{\cancel{nAe}}\hat{x} \times \vec{B}$$

$$= -Id \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix} = I d B_z \hat{y} - I d B_y \hat{z}$$

We know this must be balanced by gravity $\vec{F}_g = mg(-\hat{y})$, thus

$$B_y = B_x = 0$$

$$I d B_z = mg$$

$$B_z = \frac{mg}{Id}$$

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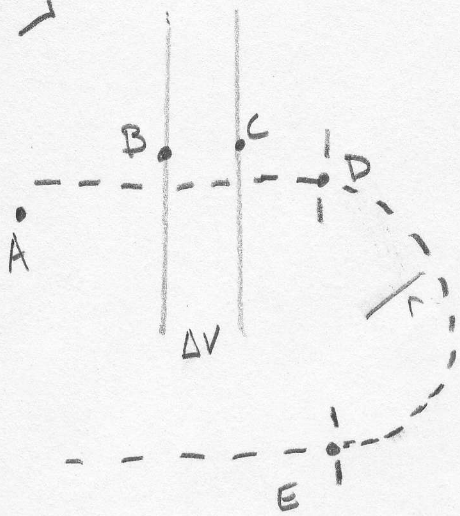

$r = d/2$

At A, $V_A << 1$, very low speed,

Between B → C the ion travels through ΔV, by conservation of energy

$$\tfrac{1}{2} M V_c^2 = q \Delta V$$

$$V_c = \sqrt{\frac{q}{m} 2\Delta V}$$

No change from C → D, thus it enters loop with $V_D = V_c$

$$\omega = \frac{|q|\,|\vec{B}|}{\gamma M} \approx \frac{q\,|\vec{B}|}{m} \qquad V_D = \omega r$$

hence

$$\frac{qr}{m} |\vec{B}| = \sqrt{\frac{q}{m} 2\Delta V}$$

$$|\vec{B}| = \frac{m}{qr} \sqrt{\frac{q}{m} 2\Delta V}$$

$$= \frac{1}{r} \sqrt{\frac{m}{q} 2\Delta V}$$

As an aside, since the $\vec{B}$ field did no work $V_D = V_E$ only in opp. direction

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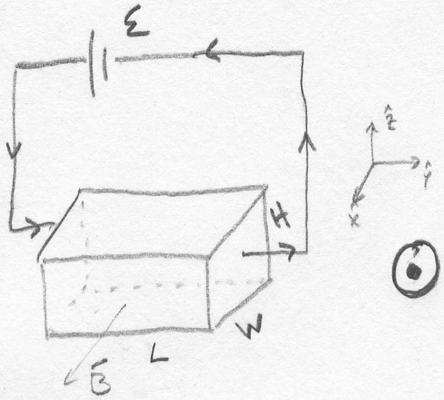


Along the bar current flows along $\hat{y}$

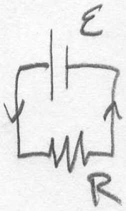
$\odot \quad \vec{B} = 1.8\,T\,\hat{x}$

Solve for R first, n, υ are known thus

$$R = \frac{L}{\sigma A} = \frac{L}{\sigma A} \qquad \sigma = en\upsilon$$

$$= \frac{L}{en\upsilon A}$$

the circut looks like



hence the current is

$$\varepsilon - IR = 0$$

$$I = \varepsilon / R$$

$$\Delta V_{hall} = \frac{I}{enA} BH = \frac{IBH}{\cancel{enA}}\, \varepsilon\, \frac{\cancel{en}\upsilon \cancel{A}}{L} = \frac{H}{L} \varepsilon B \upsilon$$