Field lines and Path Integrals

We are going to write a program that plots the exact field lines of a dipole, on and off the axis. In this assignment we are going to be using reduced units, i.e. set $\frac{q}{4\pi\epsilon_0} = 1$, s = 0.1 where q is the charge and s is the atomic dipole separation. The center of the dipole will be in the center of the coordinate system with the charges on the x-axis. To help you code this assignment, first write the electric field (it's a vector!) down on a piece of paper. Do not simplify this expression, as we are plotting the exact expression for this assignment,

Now create a grid of points over the interval (-1,-1): (1,1), that is create an array of x and y coordinates (call them X,Y). For each point on that grid you will need to use your expression for the electric field. Save its components in the arrays Ex, Ey. You can plot these vector field lines with:

from pylab import * quiver(X,Y,Ex,Ey, scale=150) show()

Verify that the field lines drawn match your expectations before you move on.

Next we want to show, numerically the path Independence of the potential. We are going to integrate around a closed loop:

$$\Delta V = \oint \vec{E} \cdot \vec{dl} \tag{1}$$

The path that you are going to take is a square ABCDA defined by:

$$\vec{A} = \langle 0.75, 0.75, .0 \rangle, \vec{B} = \langle -0.75, 0.75, .0 \rangle, \vec{C} = \langle -0.75, -0.75, .0 \rangle, \vec{D} = \langle 0.75, -0.75, .0 \rangle$$
(2)

If you remember anything from chapter 16, the answer to this should be obvious. Nonetheless I would like you to compute the solution numerically. Since the path is a square, $d\vec{l}$ is going to be $\pm \hat{x}$ or $\pm \hat{y}$ making the actual integral especially easy to compute.