PHYSICS 480-003 Contemporary Physics II – Homework 1

In all problems involving solutions to problems involving equations, leave your solution in "equation" form until the last step. You may ask each other questions (and i encourage you to do so, the undergrad. student lounge is there just for you!) but you are expected to all do your own work!

Define:

$$\begin{aligned}
\vec{r}_1 &= (1.0, 2.0, 3.0) \quad (1)\\
\vec{r}_2 &= (2.0, 2.0, 0.0) \quad (2)\\
\vec{r}_3 &= (3.0, 2.0, 1.0) \quad (3)\\
\vec{r}_4 &= (1.0, 1.0, 1.0) \quad (4)\\
\vec{r}_5 &= (1.0, 1.0, 2.0) \quad (5)
\end{aligned}$$

1. Vector math review
   (a) What is $|\vec{r}_2|$
   (b) What is $\vec{r}_5 \times \vec{r}_3$ (Cross Product)
   (c) What is the angle between $\vec{r}_5$ and $\vec{r}_4$

2. Show algebraically (from the definition of cross product) that

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \tag{6}$$

3. Show algebraically (from the definition of cross product) that

$$\vec{A} \times c\vec{B} = c(\vec{A} \times \vec{B}) \tag{7}$$

4. Show algebraically (from the definition of dot product) that

$$d\frac{(\vec{A} \cdot \vec{B})}{dx} = \frac{d\vec{A}}{dx} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dx} \tag{8}$$

5. A large merry go round with radius R and mass M is mounted on an axle so as to rotate freely. A small child with mass m running with velocity $\vec{v}$ runs towards the merry ground (with a velocity of (v,0,0) m/s) and jumps on at the point (0,R,0) m . (The axis of the merry-go-round is at (0,0,0)). Assume that the merry-go-round was rotating initially at an angular velocity of ω_0. Assume when I say "what is the angular momentum" i mean with respect to the merry-go-round axis.
   (a) What is the angular momentum of just the child 10 seconds before the child jumps on the merry go round?

(b) What is the angular momentum of just the child right before the child jumps onto the merry go round?

(c) What is the moment of inertia I of the merry go round?

(d) What is the angular momentum of the merry go round right before the child jumps onto the merry go round?

(e) What is the angular momentum of the child and the merry go round right before the child jumps onto the merry go round?

(f) What is the angular momentum of the child and the merry go round right after the child jumps onto the merry go round?

(g) What is the moment of total moment of inertia of the the child and the merry go round after the child jumps on the merry go round?

(h) Given this, derive a formula for the what the total angular velocity of the merry go round (with the child on it) after the child jumps on!

(i) Suppose we had picked a different spot say (1,0,0) to calculate the angular momentum around for our analysis (i.e. before and after). Would the angular momentum of the system be conserved before and after? Explain clearly why or why not!

I have broken this problem down in parts for you. Basically every problem that follows is a variation of the theme. Your system will either conserve angular momentum or not. If it does not there is a change in the angular momentum because of an external force (i.e. torque) which will allow you to calculate the change in angular momentum by integrating. Break down each problem down into small doable steps like above and don't panic :))

6. It has been surmised that the sun was formed in the gravitational collapse of a dust cloud which filled the space now occupied by the solar system and beyond. If we assume that the original cloud was a uniform sphere of radius R_0 with an average angular velocity of ω_0, how fast should the sun be rotating now? For the purposes of this problem, you may ignore the relatively small masses of the planets and assume the sun is a uniform sphere of radius R_s

Problem 9.2, 9.3, 9.5 ,9.9, 9.11, 9.20 , 9.23