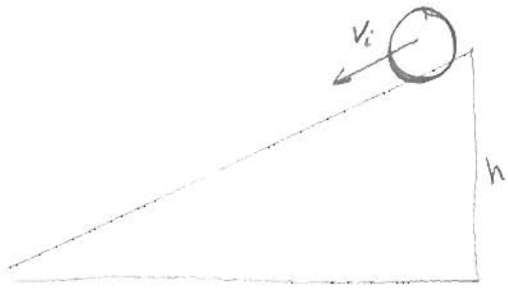
PHYS 113

HW 8 sol: Travis Hoppe

#7.3] a]



Energy is conserved

$$\tfrac{1}{2} m v_i^2 + \tfrac{1}{2} I \omega_i^2 + mgh = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I \omega_f^2$$

rolling w/o slipping $\rightarrow$ $v = \omega r$

hoop has $I = mR^2$

$$\tfrac{1}{2} m V_i^2 + \tfrac{1}{2} m R^2 \left(\frac{V_i^2}{R^2}\right) + mgh = \tfrac{1}{2} m V_f^2 + \tfrac{1}{2} m \cancel{R^2} \left(\frac{V_f^2}{\cancel{R^2}}\right)$$

$$V_i^2 + gh = V_f^2$$

$$\sqrt{V_i^2 + gh} = V_f$$

b]

total mass $= m + M$

$$I = MR^2 + \cancel{m(0)^2}^{\,0} = MR^2$$

$$\tfrac{1}{2}(m+M) V_i^2 + \tfrac{1}{2} M \cancel{R^2} \left(\frac{V_i^2}{\cancel{R^2}}\right) + (m+M) gh = \tfrac{1}{2}(m+M) V_f^2 + \tfrac{1}{2} R^2 M \left(\frac{V_f^2}{R^2}\right)$$

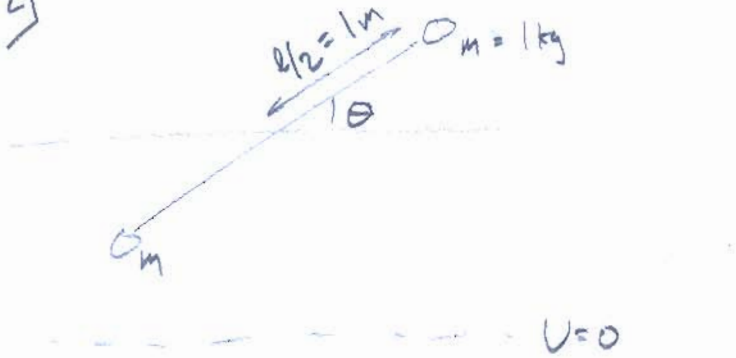
$$\frac{V_i^2}{2}\left[m + M + M\right] + (m+M) gh = \frac{V_f^2}{2}\left[m + M + M\right]$$

$$V_f = \left(\frac{V_i^2 (2M + m) + 2(m+M) gh}{2M + m}\right)^{1/2}$$

PHYS 1B

HW 8 sol. key Travis Hoppe

#2]



Since center of mass doesn't move

$$U = U_{cm} = (2m)\,g\left(\frac{l}{2}\right) = mgl \cong 19.6\,J$$

$$I = \sum m_i r_i^2 = 2m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{2} = 2\ kg\,m^2$$

After adding 40J of energy, it must be all rotational, since there is no movement of the center of mass.

$$K_{trans} = 0 \qquad K_{rot} = 40J$$

$$K_{rot} = 40J = \tfrac{1}{2} I \omega^2$$

$$\omega = \sqrt{40} \cong 6.3\ Hz$$

$$v = \omega r = \left(\sqrt{40}\right)\frac{l}{2} = 6.3\ m/s$$

PHYS 1B

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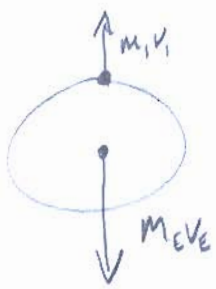
Assume:
- All people stand together in one spot right on top of the earth
- All people have a mass of 70 kg
- All people jumped up to a height of 1/2 m

conservation of energy gives $V = \sqrt{2gh} \approx 3\,\text{m/s}$

Using of conservation of momentum



$$\vec{p}_i = \vec{p}_f$$

$$0 = m_1 V_1 - M_E V_E$$

$$V_E = \frac{m_1 V_1}{M_E} = \frac{3(6\cdot 10^9)(70)}{6\cdot 10^{24}} = 2\cdot 10^{-13}\,\text{m/s}$$

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#8.6] Momentum is conserved, assume head-on col.

$$Mv + m\cancel{(0)}^{=0} = (M+m)v' \qquad v' = \frac{Mv}{m+M}$$

$$(Mv' - Mv) + (mv') = 0$$

$$\Delta p_{car} + \Delta p_{mosq} = 0$$

$$\Delta p_{mosq} = mv' = m\left(\frac{Mv}{m+M}\right) = \frac{mMv}{m+M} \approx mv$$

$$\Delta p_{car} = Mv' - Mv = M\left(\frac{Mv}{m+M} - v\right) = vM\left(\frac{M}{m+M} - 1\right)$$

$$= vM\left(\frac{-m}{m+M}\right)$$

$$= -\frac{vMm}{m+M} \approx -vm$$

since $M \gg m$

The force on the mosquito = force on car

However $a_{mos} \neq a_{car}$, the mosquito faces a <u>far</u> larger deacc. than the car - hence he is squished!

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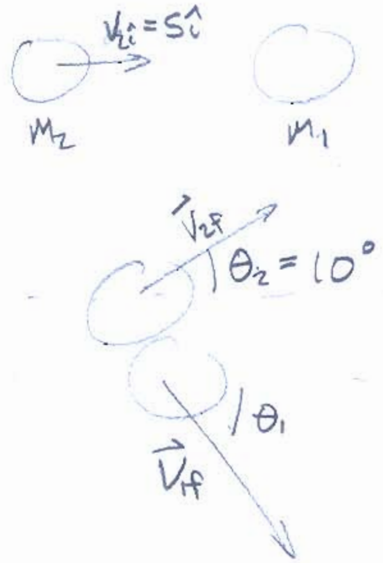
#5]



Elastic col. thus both momentum & energy are conse

$$\tfrac{1}{2} m_2 V_{2i}^2 = \tfrac{1}{2} m_1 V_{1f}^2 + \tfrac{1}{2} m_2 V_{2f}^2$$

$$V_{2i}^2 = (m_1/m_2) V_{1f}^2 + V_{2f}^2$$

$\vec{P}_i = \vec{P}_f$ gives two eq
one in $\hat{i}$ and
one in $\hat{j}$

$$\vec{V}_{2f} = V_{2f} \left(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j} \right)$$
$$\vec{V}_{1f} = V_{1f} \left(\cos\theta_1 \hat{i} - \sin\theta_1 \hat{j} \right)$$

$$m_2 V_{2i} = m_2 V_{2f} \cos\theta_2 + m_1 V_{1f} \cos\theta_1$$

$$0 = m_2 V_{2f} \sin\theta_2 - m_1 V_{1f} \sin\theta_1$$

$$V_{2f} = \frac{V_{1f}}{\sin\theta_2} \sin\theta_1 \, \frac{m_1}{m_2}$$

Solving the 3 eq. in MAPLE gives

$$V_{1f} = .43 \text{ m/s} \quad V_{2f} = 4.97 \text{ m/s}$$
$$\theta_1 = 82.5°$$