PHYS 431/531: Galactic Astrophysics

Fall 2021, Homework #5

(Due November 13, 2021)

1. Assuming an average stellar mass of $0.5\,M_\odot$ and $\Lambda = r_c/1\,\mathrm{AU}$, use the information in Table 3.1 of Sparke and Gallagher to estimate the relaxation time $t_{\rm relax}$ at the center of the globular cluster 47 Tucanae. Show that the crossing time $t_{\rm cross} \approx 2r_c/\sigma_r$ is approximately $10^{-3}\,t_{\rm relax}$. (Sparke & Gallagher problem 3.16.)

2. The velocities of stars in a stellar system are described by a three-dimensional Maxwellian distribution—that is, the probability of a star having speed between v and $v + dv$ is $f(v)\,dv$, where
$$f(v) \;=\; Av^2\,e^{-mv^2/2kT}.$$
Here, A is a normalization constant, defined so that $\int_0^\infty f(v)dv = 1$, m is the stellar mass, assumed constant, k is Boltzmann's constant, and T is the temperature of the system. Verify the statement made in class that the mean stellar kinetic energy is $\frac{1}{2}m\langle v^2\rangle \;=\; \frac{3}{2}kT$.

3. Two extrasolar "comets" were observed to pass through the inner solar system during a 2-year period. Assuming that they are members of a roughly uniform interstellar background of escaped planetesimals, with typical velocities relative to the Sun of $V = 20$ km/s, calculate the number density of this background consistent with an encounter rate of one approach within the orbit of Jupiter ($r_p = 5.2$ AU) per year.

4. Work out the details of the simple evaporative model discussed in class. Stars evaporate from a cluster of mass M on a time scale $t_{ev} = \alpha t_R$, where $\alpha \gg 1$, so
$$\frac{dM}{dt} = -\frac{M}{\alpha t_R}\,. \tag{1}$$
For pure evaporation, each escaping star carries off exactly zero energy (i.e. stars barely escape the cluster potential), so the total energy of the cluster remains constant.

   (a) If the cluster potential energy can always be written as $U = -k\frac{GM^2}{2R}$ for fixed k, where R is a characteristic cluster radius, and assuming that the cluster is always in virial equilibrium, show that $R \propto M^2$ as the cluster evolves.

   (b) Assuming that the relaxation time t_R scales as $M^{1/2}R^{3/2}$, so
$$t_R = t_{R0}\left(\frac{M}{M_0}\right)^{1/2}\left(\frac{R}{R_0}\right)^{3/2},$$
solve equation (1) to determine the lifetime of the cluster (in terms of its initial relaxation time t_{R0}). Also write down an expression for the mean cluster density as a function of time.

   (c) Estimate this lifetime for a globular cluster of mass $5 \times 10^5 M_\odot$, radius 10 pc, and mean stellar mass $0.5M_\odot$.