PHYS 431/531: Galactic Astrophysics

Fall 2021, Homework #4

(Due November 6, 2021)

1. Air at sea level on Earth's equator has density $\rho = 1.1$ kg/m^3 and sound speed $c_s = 345$ m/s.

   (a) What is its Jeans length? What is the Jeans mass?

   (b) By how much does the self-gravity of air change the frequency of a sound wave of wavelength 10 m?

   (c) By what fraction (relative to the "short-wavelength" result $f = c_s/\lambda$) does the self-gravity of an interstellar cloud change the frequency of a sound wave with wavelength equal to one quarter the Jeans length?

2. Estimate the masses of star clusters having

   (a) root mean square velocity 15 km/s and half-mass radius 5 pc,

   (b) mean density 1000 pc^{-3}, rms velocity 2 km/s, and mean stellar mass $0.5 M_\odot$,

   (c) dynamical time 10^6 yr and radius 10 pc.

3. A self-gravitating gas sphere of radius R is in hydrostatic equilibrium, so its internal pressure gradient balances the acceleration due to gravity: $dP/dr = -G\rho(r)M(r)/r^2$, where $P(r), \rho(r)$, and $M(r)$ are, respectively, the pressure, the density, and the mass interior to radius r.

   (a) For an ideal gas equation of state $P = \rho kT/\mu$, show that the total thermal energy of the star is $E_T = 6\pi \int_0^R r^2 P(r)\, dr$.

   (b) Integrate this equation by parts, assuming $P(R) \approx 0$, and use the equation of hydrostatic equilibrium to show that the sphere satisfies the virial theorem: $2E_T + U = 0$, where U is the total gravitational potential energy.

4. Interstellar gas in many galaxies is in virial equilibrium with the stars, in that the rms speed of the gas particles is the same as the rms stellar speed. Consider a large elliptical galaxy with a virial radius of 150 kpc and a mass of 5×10^{12} solar masses. Calculate the rms stellar velocity using the virial theorem. Hence estimate the temperature of the interstellar gas, assuming that it is composed entirely of hydrogen.