PHYS 431/531: Galactic Astrophysics

Fall 2021, Homework #2

(Due October 8, 2021)

1. (a) Use Gauss's law to derive an expression for the gravitational force in the z direction due to an infinite sheet of mass of surface density Σ lying in the x–y plane.

   (b) A certain star has velocity 30 km/s perpendicular to the Galactic plane as it crosses the plane, and is observed to have a maximum departure above the plane of 500 pc. Approximating the disk as an infinite gravitating sheet of matter, estimate its surface density Σ (i) in $\mathrm{kg\,m^{-2}}$ and (ii) in $M_\odot\,\mathrm{pc}^{-2}$.

2. A simple axisymmetric model of the stellar number density $n(R, z)$ in the Galactic disk is

$$n(R, z) = n_0\, e^{-R/h_R}\, e^{-|z|/h_z},$$

   where R is distance from the Galactic center, z is distance from the disk plane, and h_R and h_z are (constant) scale heights.

   (a) If all stars have the same luminosity L_*, integrate the above expression with respect to z to determine the disk surface brightness $\Sigma(R)$ (that is, the total luminosity per unit area at any given location).

   (b) Now integrate Σ with respect to R to determine the total luminosity L_G of the Galaxy.

   (c) If $L_G = 2 \times 10^{10}\, L_\odot$, and $h_R = 4$ kpc, what is the local surface brightness in the vicinity of the Sun, at $R = 8$ kpc?

   (d) If $h_z = 250$ pc and $L_* = L_\odot$, calculate the local density of stars in the solar neighborhood (at $z = 0$).

3. (a) Given the definitions of the Oort constants A and B presented in class (Eqs. 2.13 and 2.16 in the text),

$$A = -\tfrac{1}{2}\, R \left(\frac{V}{R}\right)'\Bigg|_{R=R_0} \qquad B = -\tfrac{1}{2}\, \frac{(RV)'}{R}\Bigg|_{R=R_0},$$

   verify that $A + B = -V'(R_0)$ and $A - B = V_0/R_0$, where $V(R)$ is the Galactic rotation law, R_0 is the distance from the Sun to the Galactic center, and $V_0 = V(R_0)$.

   (b) Using the GAIA measurements of $A = 15.1\pm0.1$ km/s/kpc and $B = -13.4\pm0.1$ km/s/kpc, and taking $R_0 = 8$ kpc, write down an estimate of V_0.

   (c) Consider the spherically symmetric density distribution ρ given by

$$\rho(R) = \rho_0 \left(1 + \frac{R^2}{a^2}\right)^{-1}.$$

   Derive an expression for the mass inside radius R. What is the circular orbital speed $V(R)$ at radius R? Hence determine the form of $A(R)$ and $B(R)$ for $R \gg a$.

4. If our Galaxy has a flat rotation curve with $V(R) = V_0 = 220$ km/s and the total luminosity of the disk is as in Problem 2, what is the Galactic mass to light ratio M/L inside (a) the solar circle ($R_0 = 8$ kpc), (b) $10R_0$? Compare these with the mass to light ratio of a Salpeter stellar mass distribution (see Homework 1, Problem 5) with $M_l = 0.2M_\odot$, $M_u = 100M_\odot$.

5. The star S0-102 is observed to orbit the Galactic center with an orbital period of 11.5 years. Its (properly deprojected) minimum and maximum speeds relative to the central black hole are 920 and 4830 km/s. As discussed in class, write down equations expressing conservation of angular momentum and energy at the two turning points of the orbit, and hence determine the semimajor axis and eccentricity of S0-102's orbit and the mass of the black hole.