PHYS 431/531: Galactic Astrophysics

Fall 2021, Homework #1

(Due September 30, 2021)

1. (a) The GAIA satellite observes a star to have parallax $p = (750 \pm 12) \times 10^{-6}$ arcsec. What is the most likely distance to the star, in parsecs? Estimate the spread in distance consistent with the observational limits. Explain your reasoning.

   (b) GAIA measures the star's apparent magnitude as $m_G = 12.5 \pm 0.01$. What is the best estimate of the star's absolute "G" magnitude, M_G? Estimate the possible range in M_G, given the uncertainties in both distance and apparent magnitude. Explain your reasoning.

2. (a) A distant unresolved binary consists of two stars each of apparent magnitude m. What is the apparent magnitude of the binary?

   (b) A star has apparent magnitude $m_V = 12$ and is determined spectroscopically to be an A0 main sequence star. What is its distance? (See Sparke & Gallagher Table 1.4.)

   (c) A Type Ia supernova has peak absolute magnitude $M_V = -19.3$ and is observed to have apparent magnitude $m_V = 15$ and redshift 0.018. Use these numbers to estimate the Hubble constant, H_0.

3. Assume that the Galaxy is 10 Gyr old, the rate of star formation in the past was proportional to $e^{-t/T}$, where t is time since the Galaxy formed and $T = 3$ Gyr, and stellar lifetimes are given by

   $$t(M) \ = \ 10\,\mathrm{Gyr}\,\left(\frac{M}{M_\odot}\right)^{-3}.$$

   Calculate the fractions of all (a) $0.5M_\odot$, (b) $2M_\odot$, and (c) $4M_\odot$ stars ever formed that are still around today.

4. A visual binary happens to lie in the plane of the sky — that is, its orbital plane is exactly perpendicular to the line of sight. The parallax of its center of mass location is 0.1 arcsec.

   (a) The orbital semimajor axes of the two component stars are determined by direct observation to be 0.5 and 2.0 arcseconds. What are these, in AU?

   (b) If the binary's observed orbital period is 90 yr, use Kepler's third law to determine the mass of each star in the system.

5. (a) If the mass function for stars follows the Salpeter distribution, with

   $$\xi(M) \equiv \frac{dN}{dM} = A\,M^{-2.35}$$

   (where dN is the number of stars with masses between M and $M + dM$; see Sparke & Gallagher, p. 66), for $M_l < M < M_u$, with $M_l \ll M_u$, and the stellar mass–luminosity relation is

   $$L(M) \ \propto \ M^4\,,$$

show that the total number and total mass of stars depend mainly on M_l, while the total luminosity depends mainly on M_u. Specifically, for $M_l = 0.2M_\odot$ and $M_u = 100M_\odot$, calculate the masses M_1 and M_2 such that 50% of the total mass is contained in stars with $M < M_1$, while 50% of the total luminosity is contained in stars with $M > M_2$.

(b) *[GRADUATE STUDENTS.]* Astronomers often approximate the stellar mass function $\xi(M)$ by a Salpeter power-law with a low-mass cutoff, but the Kroupa distribution

$$\xi(M) = \begin{cases} C\,M^{-0.3} & (M < 0.1M_\odot) \\ B\,M^{-1.3} & (0.1M_\odot < M < 0.5M_\odot) \\ A\,M^{-2.35} & (M > 0.5M_\odot) \end{cases}$$

is actually a much better description [A is the same as in part (a) and the other constants B and C are chosen to ensure that ξ is continuous.] If the upper mass limit in all cases is $M_u = 100M_\odot$ and we assume the same simplified mass–luminosity relation as in part (a), what low-mass cutoff M_l must be chosen in order that the truncated power-law has the same (i) total number of stars, (ii) total mass, and (iii) total luminosity as the Kroupa distribution?