PHYS 231: Introductory Astrophysics

Winter 2020

Mid-Term Examination

February 5, 2020

Answer TWO questions.
Each question is worth 50 points.
Time allowed: 75 minutes.

Where relevant, assume a main-sequence mass–luminosity relation $L \propto M^4$ and a mass–radius relation $R \propto M^{0.7}$.

1. A binary star system consists of two components: star A and a fainter companion, star B. Assume for simplicity that the orbit is circular and we see the system edge-on.

   (a) [*5 points*] Star A is observed to have a parallax of 0.07 arcseconds. How far away is the system?

   (b) [*5 points*] The maximum angular separation of stars A and B is 0.75 arcseconds. What is the actual separation (in AU) of the two stars?

   (c) [*5 points*] The orbital period of the system is 12 years. What is its total mass?

   (d) [*15 points*] Both stars can be observed spectroscopically, and the maximum Doppler shift (relative to the average value) of the spectral lines from star B is found to be 1.5 times larger than the maximum shift for star A. What is the ratio of masses M_A/M_B? (Explain your answer.) Hence find the individual stellar masses M_A and M_B.

   (e) [*10 points*] Assuming that both stars are on the main sequence and given that the absolute magnitude of the Sun is 4.8, find the absolute magnitudes of stars A and B.

   (f) [*10 points*] Suppose that, instead of the distance you obtained in part (a), the system is 1 kpc away and your telescope is unable to resolve the two components (that is, the distance increases but the physical separation you obtained in part b stays the same). What would be the apparent magnitude of the observed combined system?

2. The $0 \leftrightarrow 1$ transition occurs between the ground state (0) and the first excited rotational state (1) of the CO molecule. The photon emitted during the transition from state 1 to state 0 has frequency 115 GHz.

   (a) [*5 points*] What is the wavelength of the emitted photon?

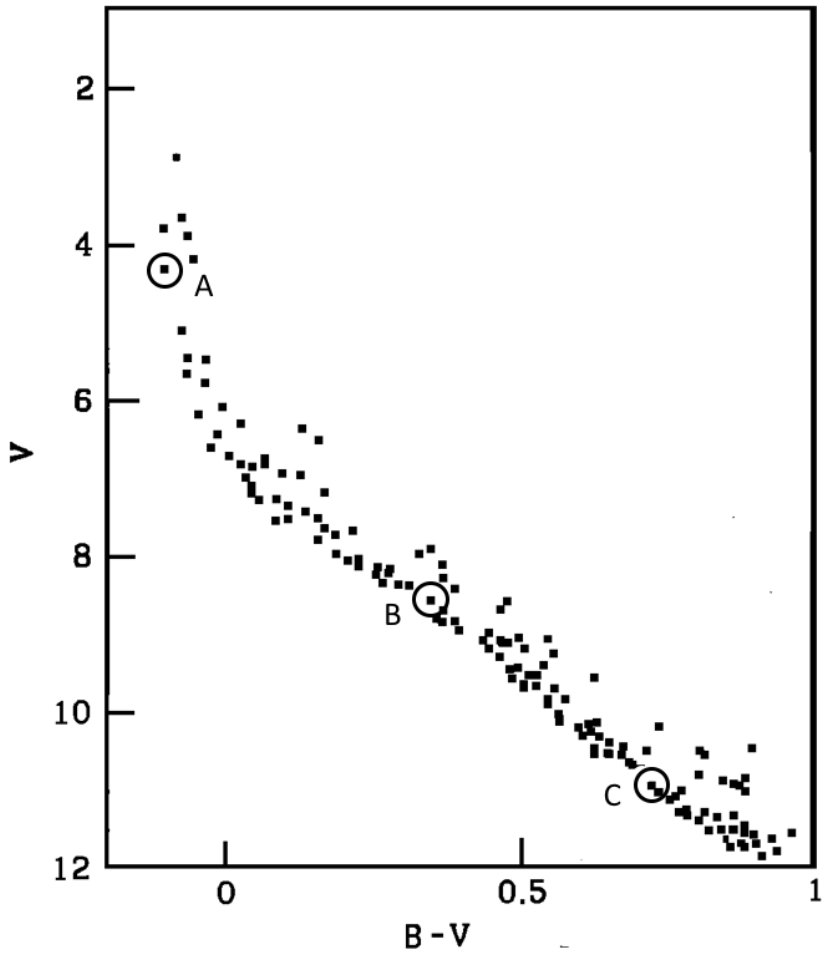
   (b) [*5 points*] What is the energy of the emitted photon?

   (c) [*5 points*] In what wavelength range (visible, UV, radio, IR, X-ray,...) does the photon lie?

   (d) [*15 points*] The atom is part of a gas cloud of temperature 20 K. What is kT in this case, and what fraction of all CO molecules will be in the upper (1) state? (The degeneracies are $g_0 = 1$, $g_1 = 3$; ignore the possibility of exciting higher energy states.)

(e) [*20 points*] The populations in part (d) are maintained by collisions among atoms, but atoms in the upper (1) state also have a probability of 7.67×10^{-8} s^{-1} of spontaneously decaying to the lower (0) state, emitting a photon in the process. If the number density of CO molecules in an interstellar cloud is $n_{CO} = 2 \times 10^{-6}\,\mathrm{m}^{-3}$, calculate the rate at which $1\,\mathrm{m}^3$ of gas emits energy due to this mechanism.

3. The Pleiades cluster is approximately 135 pc away. Its color-magnitude diagram is shown below. The vertical axis shows apparent V magnitude.



(a) [*8 points*] How much brighter (as a ratio of fluxes) is star B than star C? Be sure to write down all apparent magnitudes you estimate from the diagram.

(b) [*8 points*] Which of the three stars A, B, and C is densest? Explain your reasoning.

(c) [*8 points*] What is the absolute magnitude of star A?

(d) [*8 points*] Given that the absolute magnitude of the Sun in the V band is 4.8, what is the (V) luminosity (in solar units) of star A?

(e) [*8 points*] From the mass-luminosity relation, approximately how massive is star A?

(f) [*10 points*] Given that the main-sequence lifetime of the Sun is 10 Gyr, and assuming that star A represents the main-sequence turn-off, estimate the age of the Pleiades cluster.