PHYS 231: Introductory Astrophysics

Winter 2020

Mid-Term Solutions

1. (a) The distance is $D = 1/p$, or 14.3 pc.

   (b) The angular separation is $\theta = a/D$, so $a = D\theta = 10.7$ AU.

   (c) Kepler's third law says $P^2 = a^3/M$, where everything is measured in "solar system" units (years, AU, and solar masses). Hence the total mass is $M = a^3/P^2 = 8.5\,M_\odot$.

   (d) The Doppler shift is proportional to radial velocity, so the maximum shift in this case measures the circular orbital speed. Thus, star B is moving 1.5 times faster than star A. Relative to the center of mass, we have $M_A v_A = M_B v_B$ (where v here is speed), so $M_A/M_B = v_B/v_A = 1.5$. Since $M_A + M_B = 5.1\,M_\odot$, we have $M_A = 5.1\,M_\odot$, $M_B = 3.4\,M_\odot$.

   (e) For stars on the main sequence, $L \propto M^4$, so $L_A = 691\,L_\odot$ and $L_B = 136\,L_\odot$. Their absolute magnitudes then are $\mathcal{M}_A = 4.8 - 2.5\log_{10}(L_A/L_\odot) = -2.3$ and $\mathcal{M}_B = 4.8 - 2.5\log_{10}(L_B/L_\odot) = -0.54$.

   (e) The total luminosity is $L_{tot} = 827\,L_\odot$. At a distance of $d = 1$ kpc, this implies an apparent magnitude of $m = 4.8 - 2.5\log_{10}(L_{tot}/L_\odot) + 5\log_{10}(d/10\,\mathrm{pc}) = 7.5$.

2. (a) The frequency is $\nu = 1.15 \times 10^{11}$ Hz, so the wavelength is $\lambda = c/\nu = 2.6\,\mathrm{mm}$.

   (b) The energy of the emitted photon is $h\nu = 7.6 \times 10^{-23}$ J $= 4.8 \times 10^{-4}$ eV.

   (c) Radio.

   (d) $kT = 2.8\times10^{-22} = 3.6\,h\nu$, so from the Boltzmann formula, $n_2/n_1 = (g_2/g_1)\exp(-h\nu/kT) = 2.28$ and 69% ($= 2.28/3.28$) of all CO molecules are in the upper state.

   (e) Each excited molecule has probability $p = 7.67 \times 10^{-8}$ of emitting a photon of energy $h\nu$ in any given second, so N excited molecules will emit energy at a rate $Np\,h\nu$ W. For the given number density, a cubic meter will contain 2×10^{-6} molecules, of which 1.4×10^{-6} will be excited. Hence the emission rate is $8.1 \times 10^{-36}\,\mathrm{Wm}^{-3}$.

3. (a) I estimate V-magnitudes $m_A \approx 4.3, m_B \approx 8.5, m_C \approx 10.9$. Thus, B is 2.4 magnitudes brighter than C, which corresponds to a flux ratio of $10^{2.4/2.5} = 9.1$.

   (b) The mass-radius relation says $\bar{\rho} \propto M/R^3 \propto M^{-1.1}$, so the lowest-mass star, C, is densest.

   (c) If $m_A = 4.3$ and the distance is $D = 135$ pc, then the absolute magnitude of A is $4.3 - 5\log_{10}135 + 5 = -1.4$.

   (d) The luminosity of A relative to the Sun is then $L_A/L_\odot = 10^{(4.8-1.4)/2.5} = 290$.

   (e) The mass of star A is therefore $290^{1/4} = 4.1\,M_\odot$.

   (f) The age of the cluster is 10 Gyr$/4.1^3 = 140$ Myr. (The accepted value is about 100 Myr.)