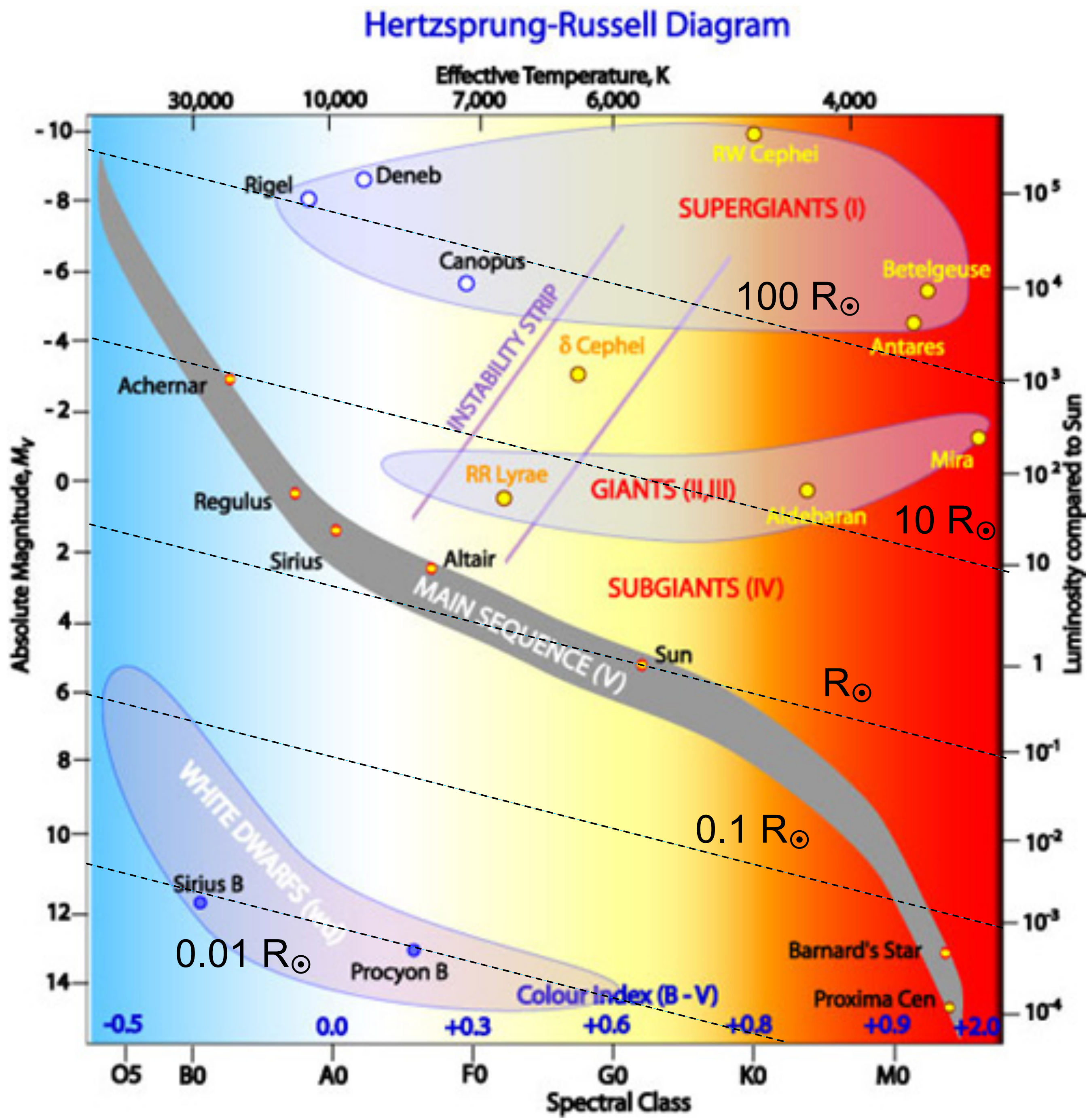
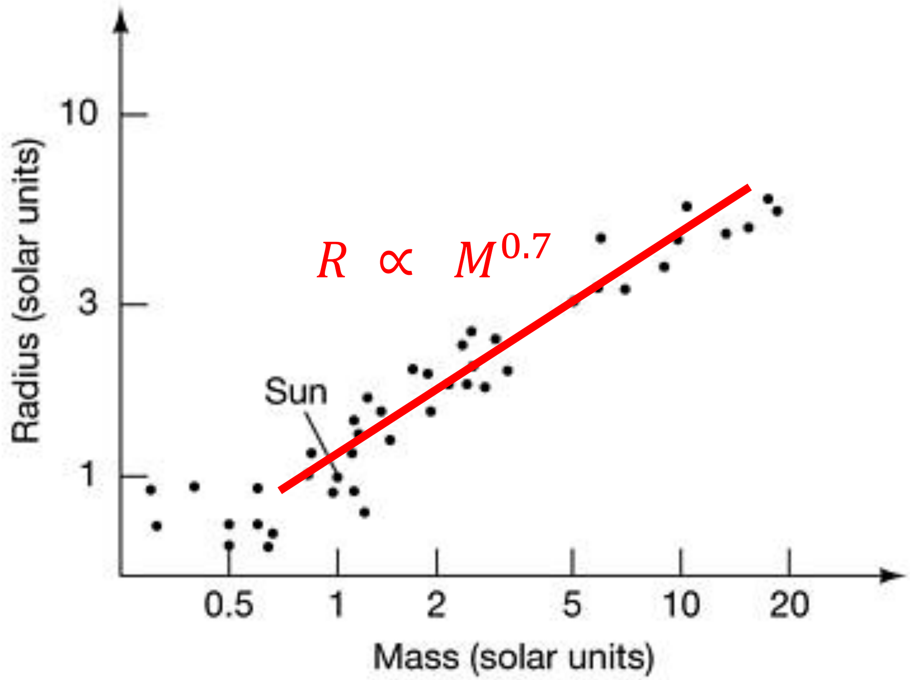
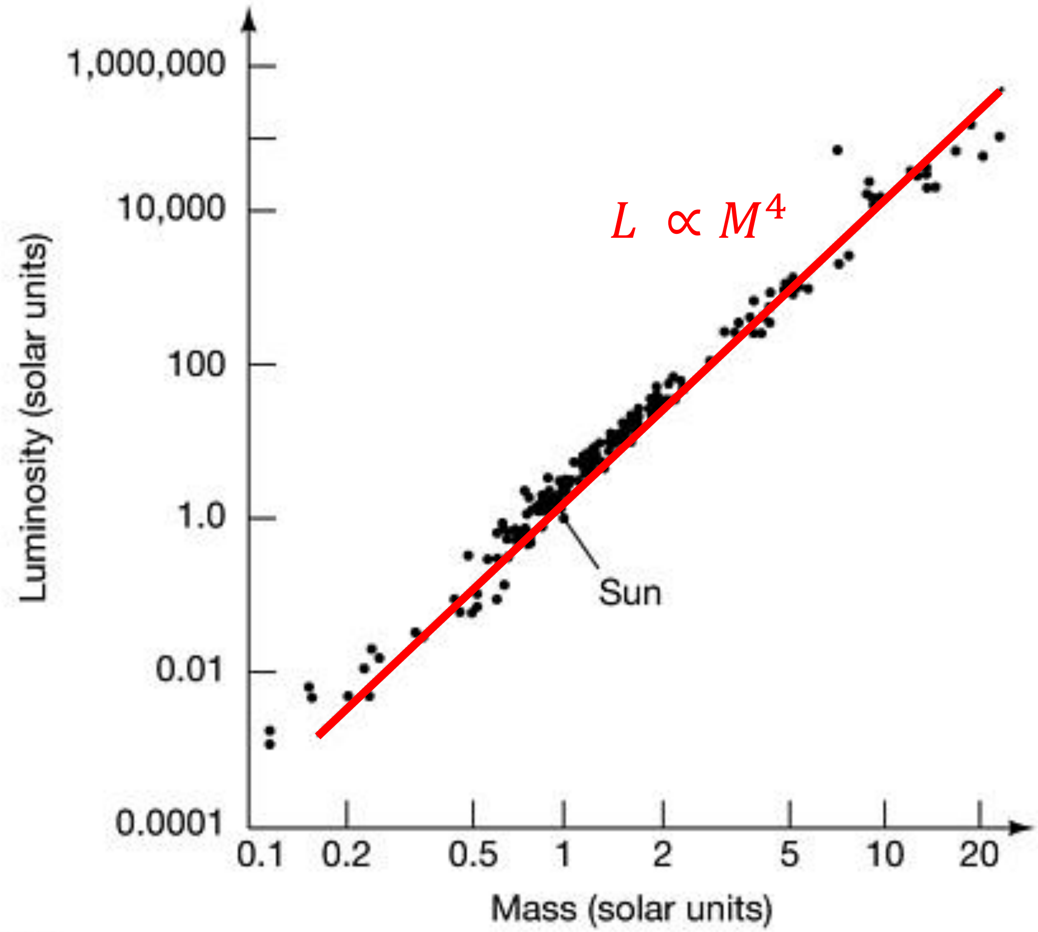


Hertzsprung-Russell Diagram
Effective Temperature, K
30,000
10,000
7,000
6,000
4,000
Absolute Magnitude, M_V
Luminosity compared to Sun
Spectral Class
O5
B0
A0
F0
G0
K0
M0
Colour Index (B - V)
-0.5
0.0
+0.3
+0.6
+0.8
+0.9
+2.0
SUPERGIANTS (I)
GIANTS (II,III)
SUBGIANTS (IV)
MAIN SEQUENCE (V)
WHITE DWARFS (wd)
INSTABILITY STRIP
Rigel
Deneb
Canopus
RW Cephei
Betelgeuse
Antares
δ Cephei
Achernar
Mira
RR Lyrae
Aldebaran
Regulus
Sirius
Altair
Sun
Sirius B
Procyon B
Barnard's Star
Proxima Cen
100 $R_\odot$
10 $R_\odot$
$R_\odot$
0.1 $R_\odot$
0.01 $R_\odot$



10
3
1
Radius (solar units)
0.5
1
2
5
10
20
Mass (solar units)
Sun
$R \propto M^{0.7}$



1,000,000
10,000
100
1.0
0.01
0.0001
Luminosity (solar units)
0.1
0.2
0.5
1
2
5
10
20
Mass (solar units)
Sun
$L \propto M^4$

$$t_{MS} \sim \frac{M}{dM/dt} \propto \frac{M}{L} \propto M^{-3}$$

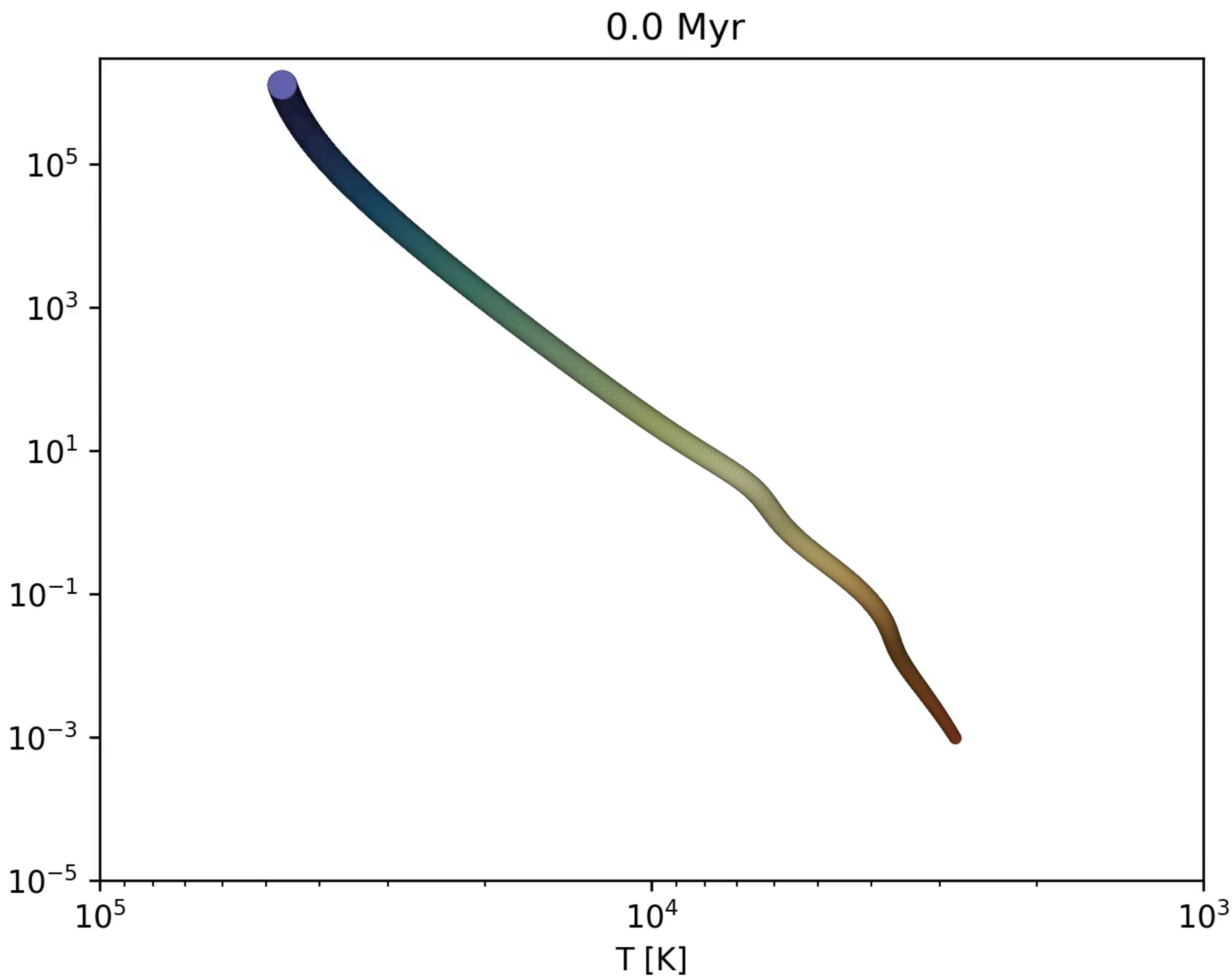
➔ stars must evolve

➔ the most massive stars evolve <u>fastest</u>!

$$t_{MS} \approx 10^{10} \left(\frac{M}{M_{\odot}} \right)^{-3} \text{ years}$$

➔ stars must evolve

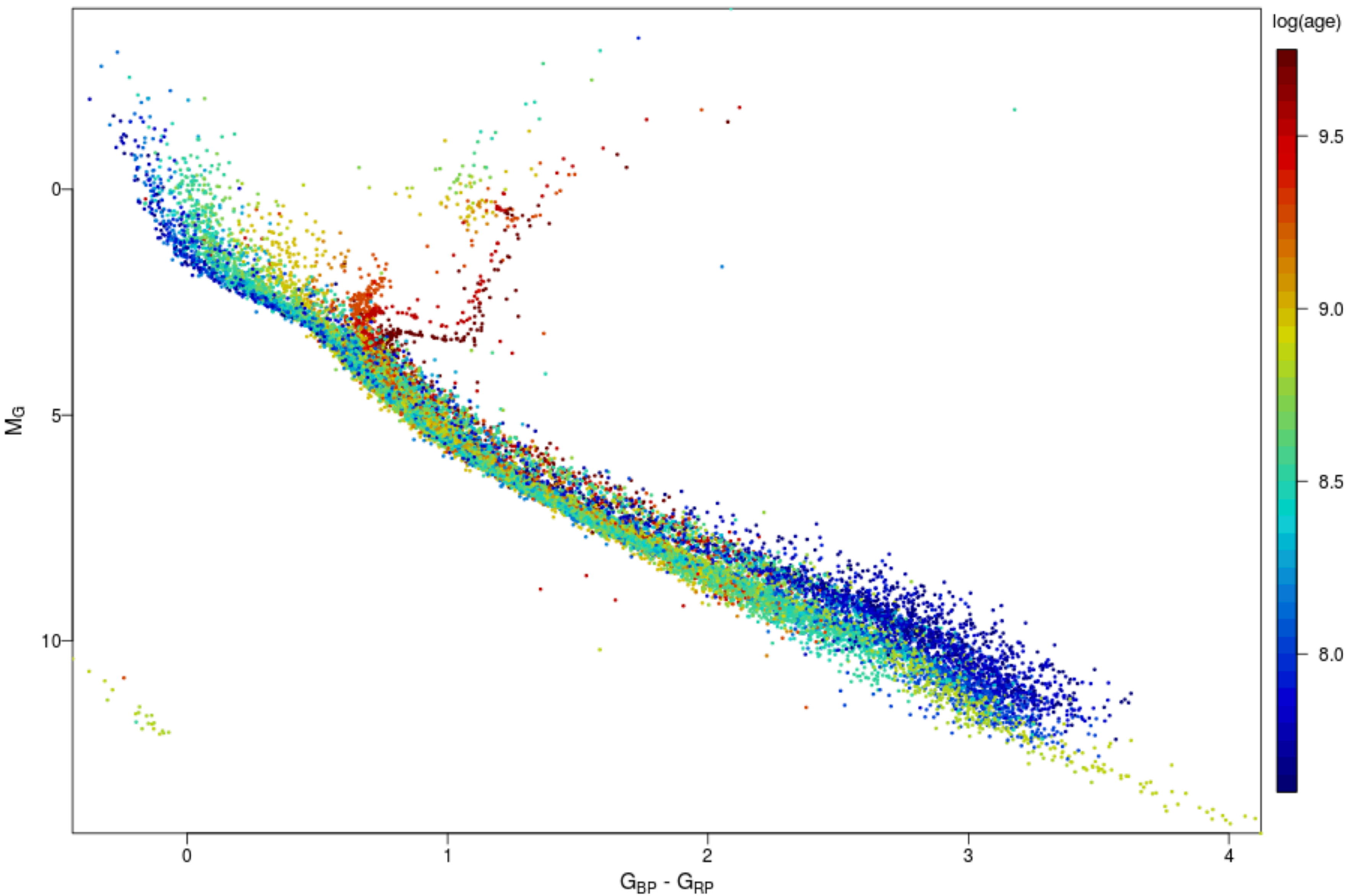
➔ the most massive stars evolve <u>fastest</u>!

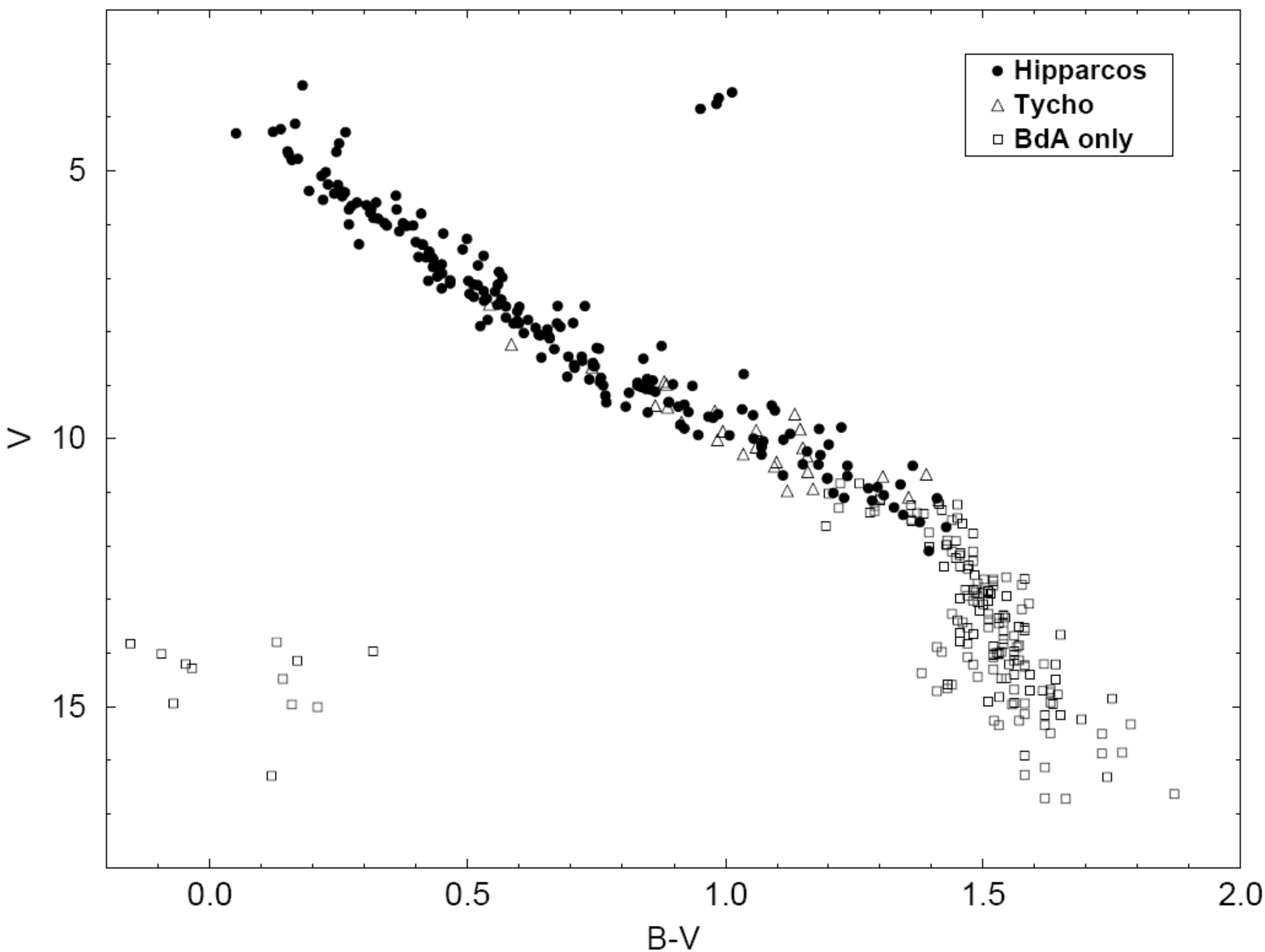


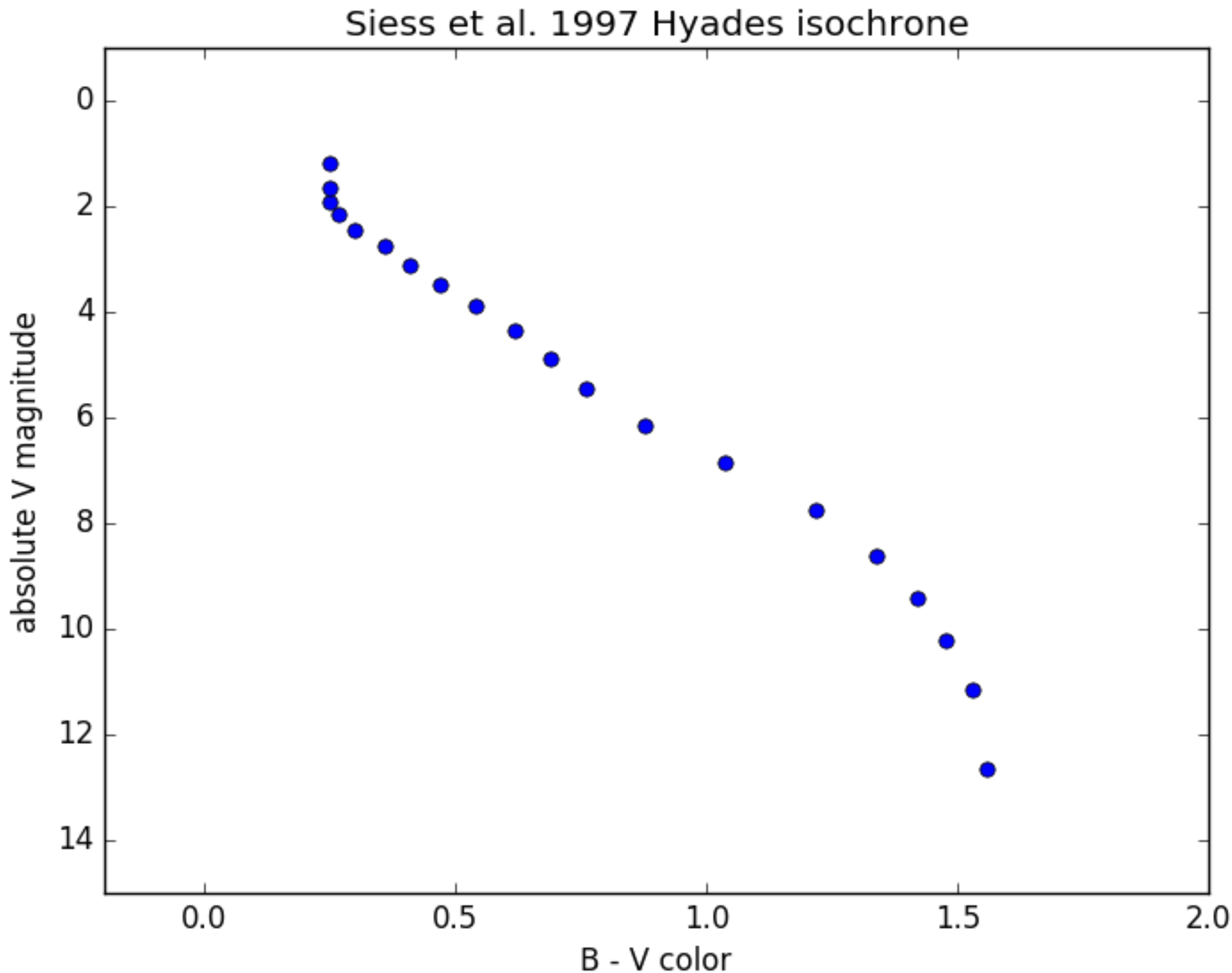
0.0 Myr
L [L$_\odot$]
10^5
10^3
10^1
10^{-1}
10^{-3}
10^{-5}
10^5
10^4
10^3
T [K]

Gaia DR2

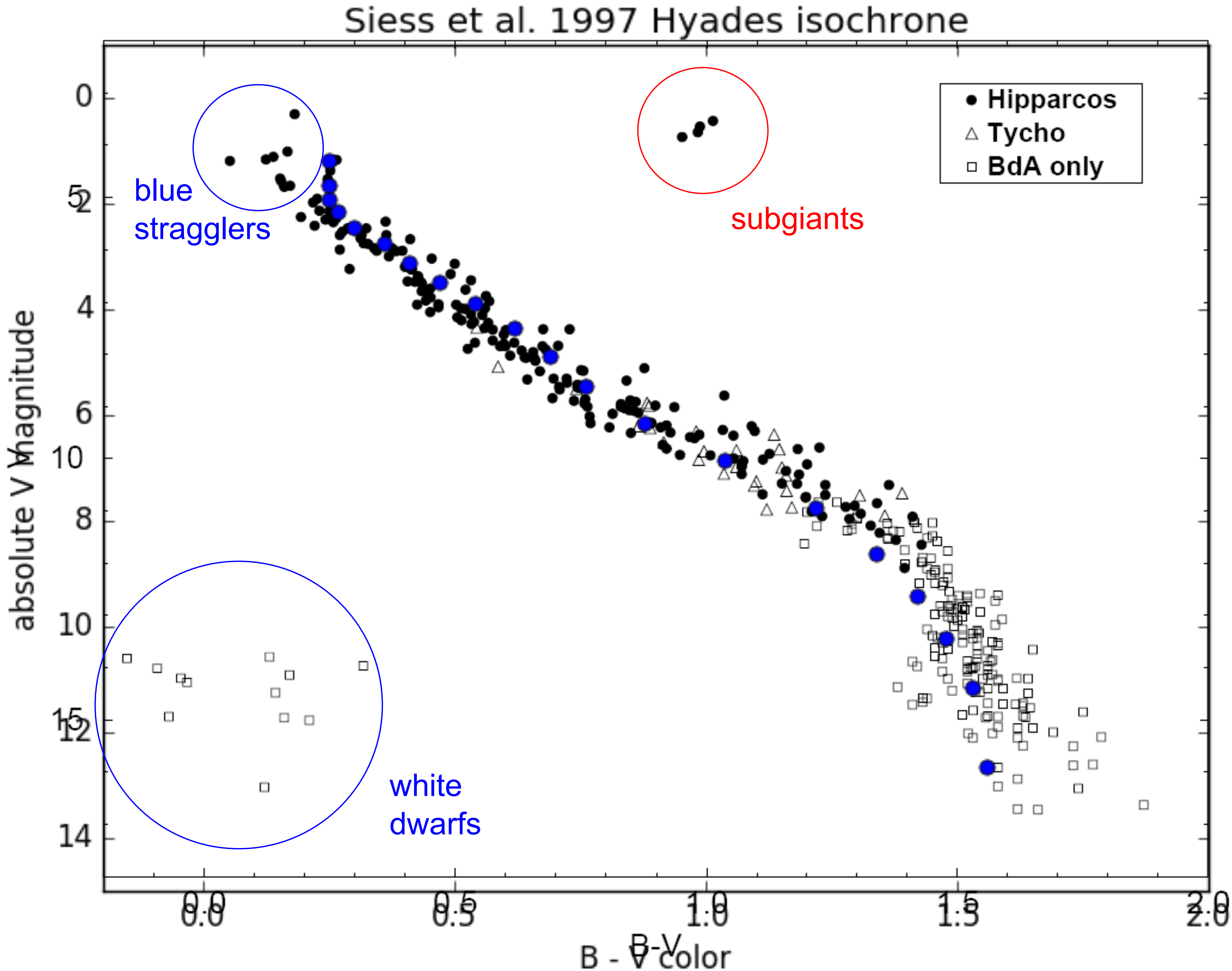
A&A 616, A10 (2018)

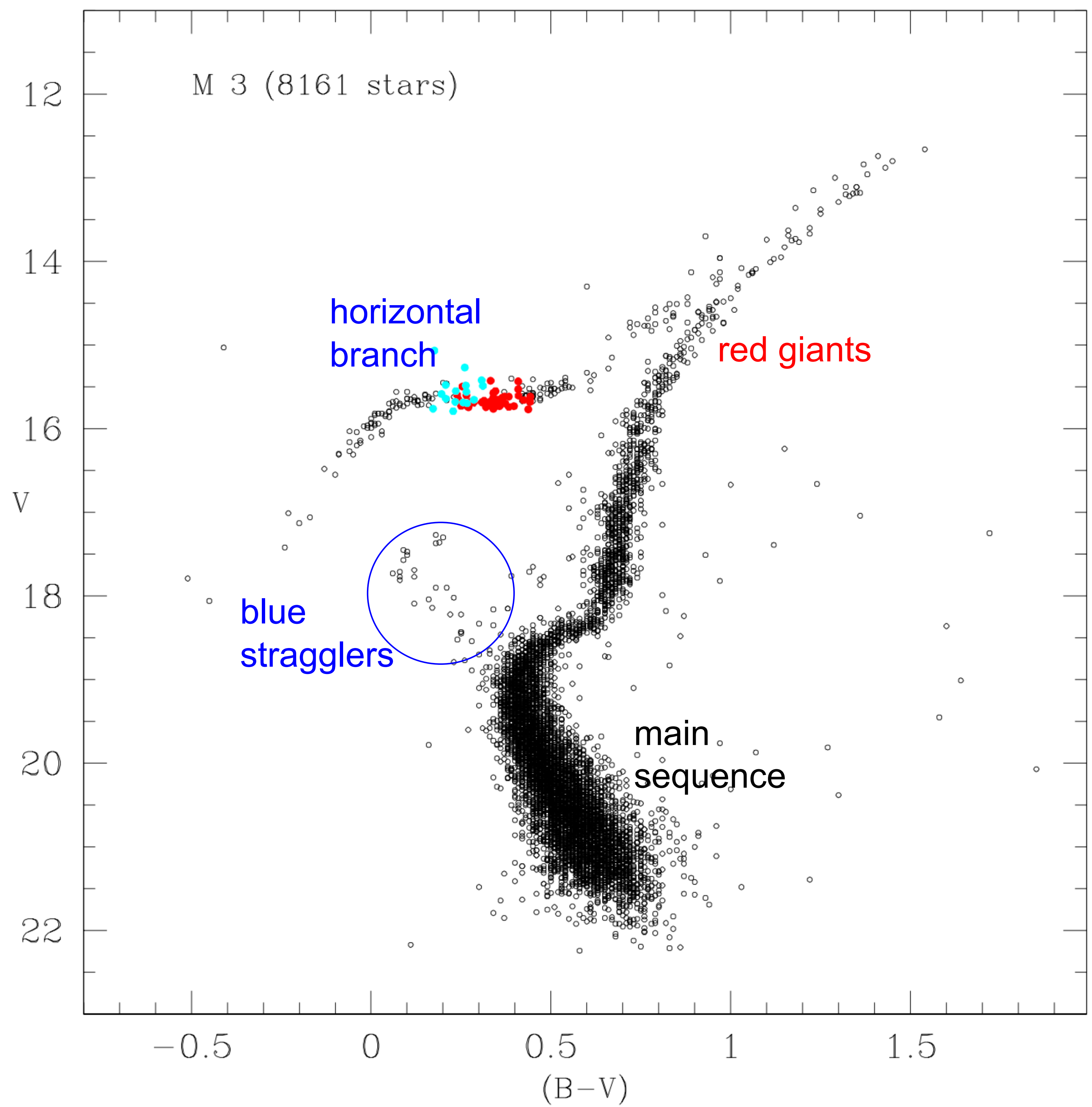


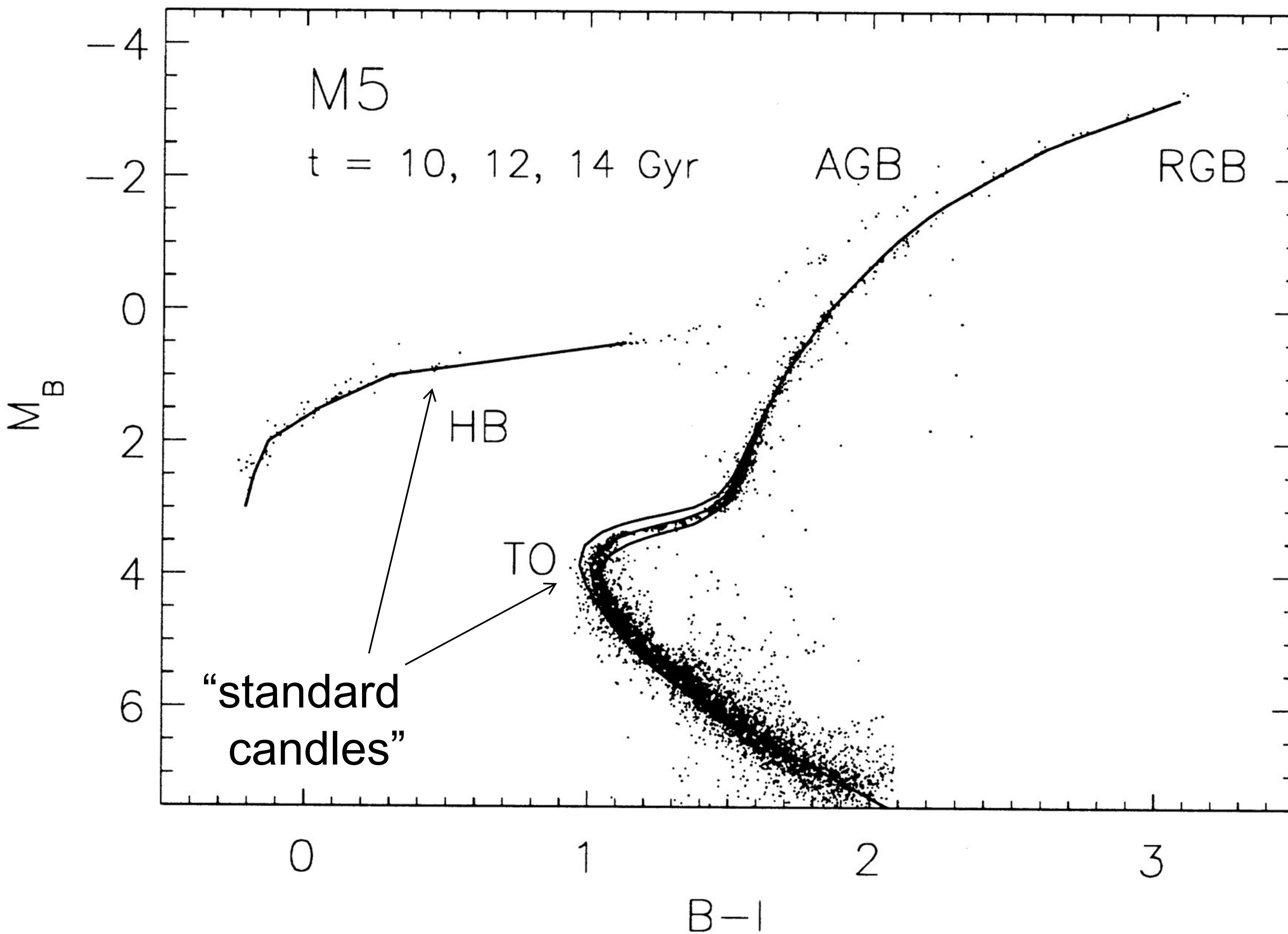

Hipparcos
Tycho
BdA only
V
5
10
15
B-V
0.0
0.5
1.0
1.5
2.0



Siess et al. 1997 Hyades isochrone
absolute V magnitude
B - V color
0
2
4
6
8
10
12
14
0.0
0.5
1.0
1.5
2.0

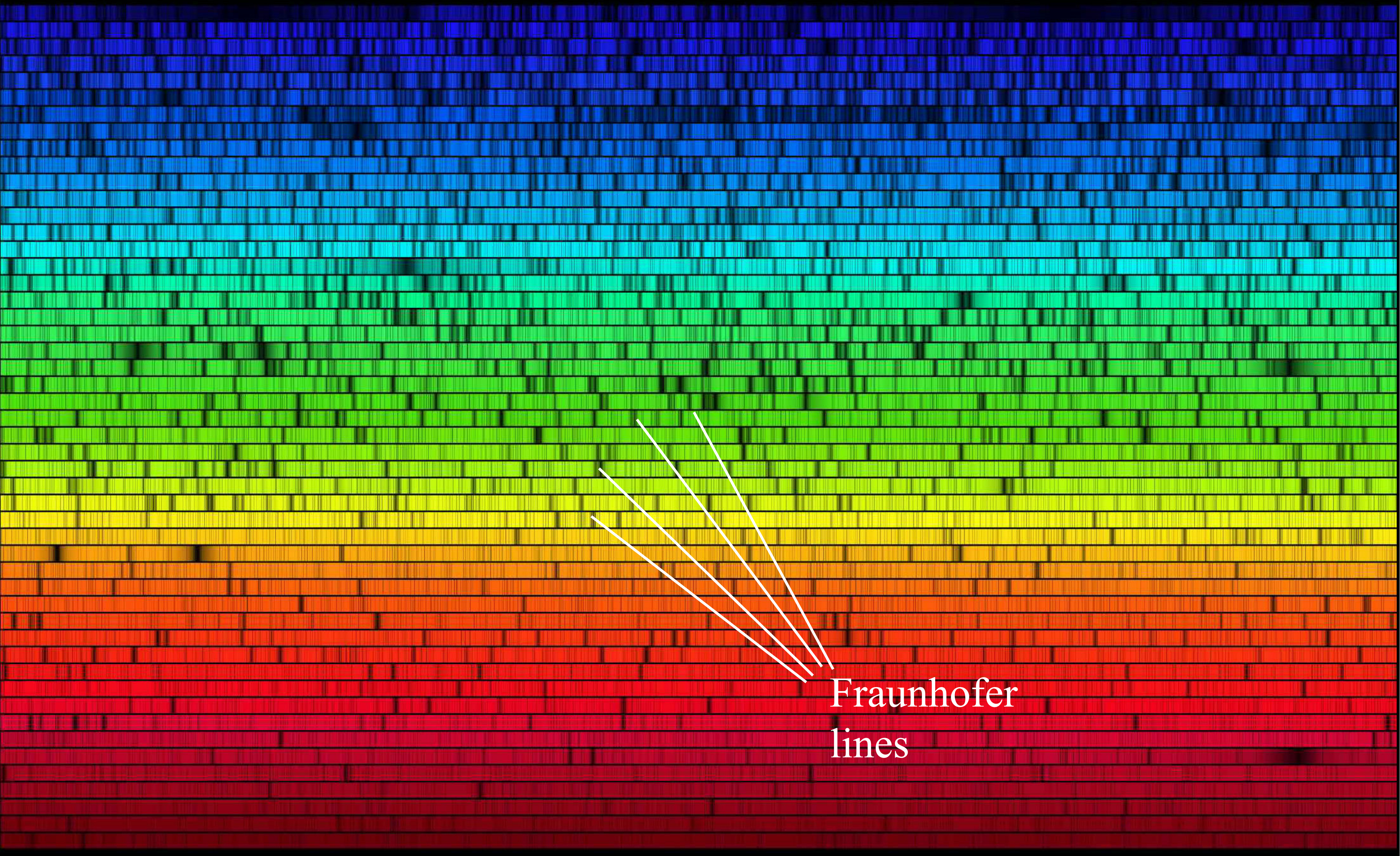

Siess et al. 1997 Hyades isochrone
Hipparcos
Tycho
BdA only
blue stragglers
subgiants
white dwarfs
absolute V magnitude
B - V color
0
2
4
6
8
10
12
14
0.0
0.5
1.0
1.5
2.0

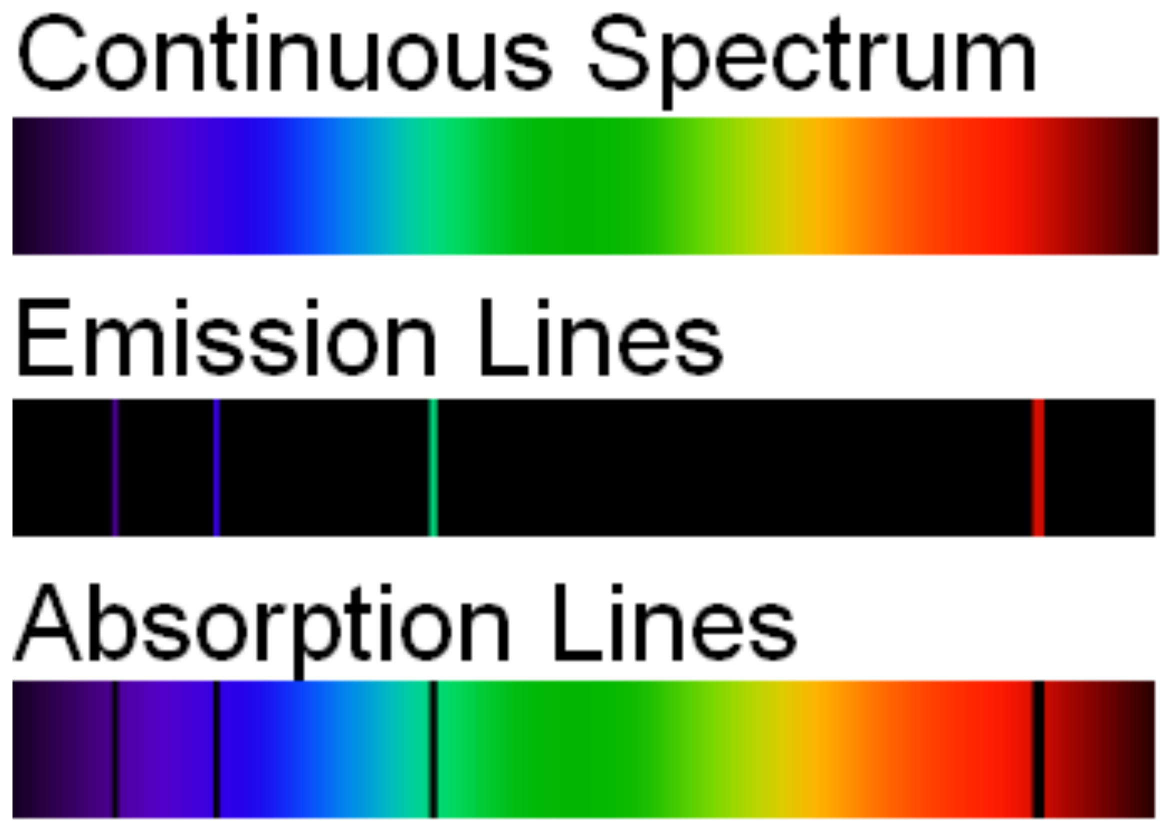

M 3 (8161 stars)
horizontal branch
red giants
blue stragglers
main sequence
V
12
14
16
18
20
22
(B−V)
−0.5
0
0.5
1
1.5

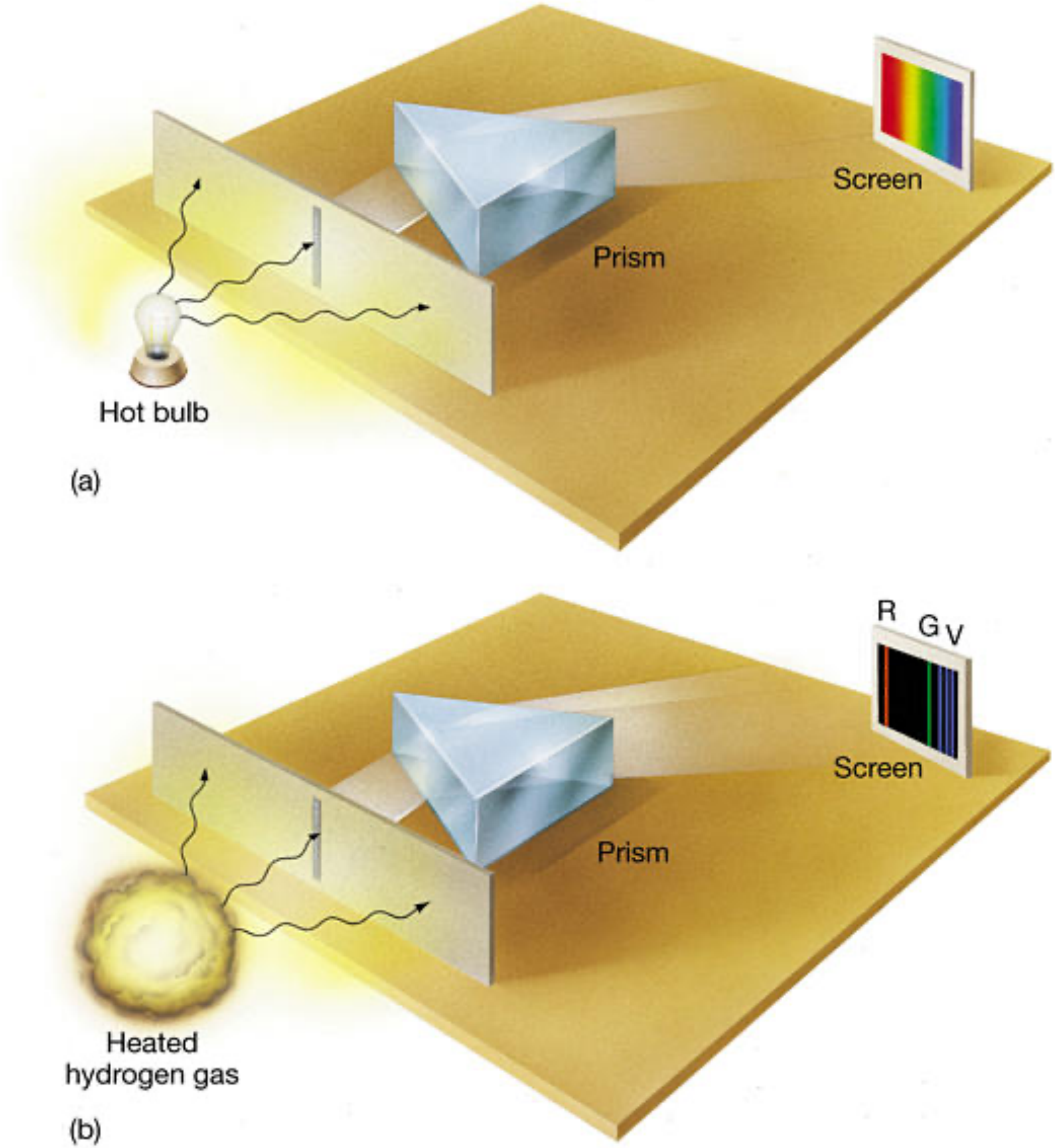


M5
t = 10, 12, 14 Gyr
AGB
RGB
HB
TO
"standard candles"
M_B
B−I
−4
−2
0
2
4
6
0
1
2
3

Solar Spectrum

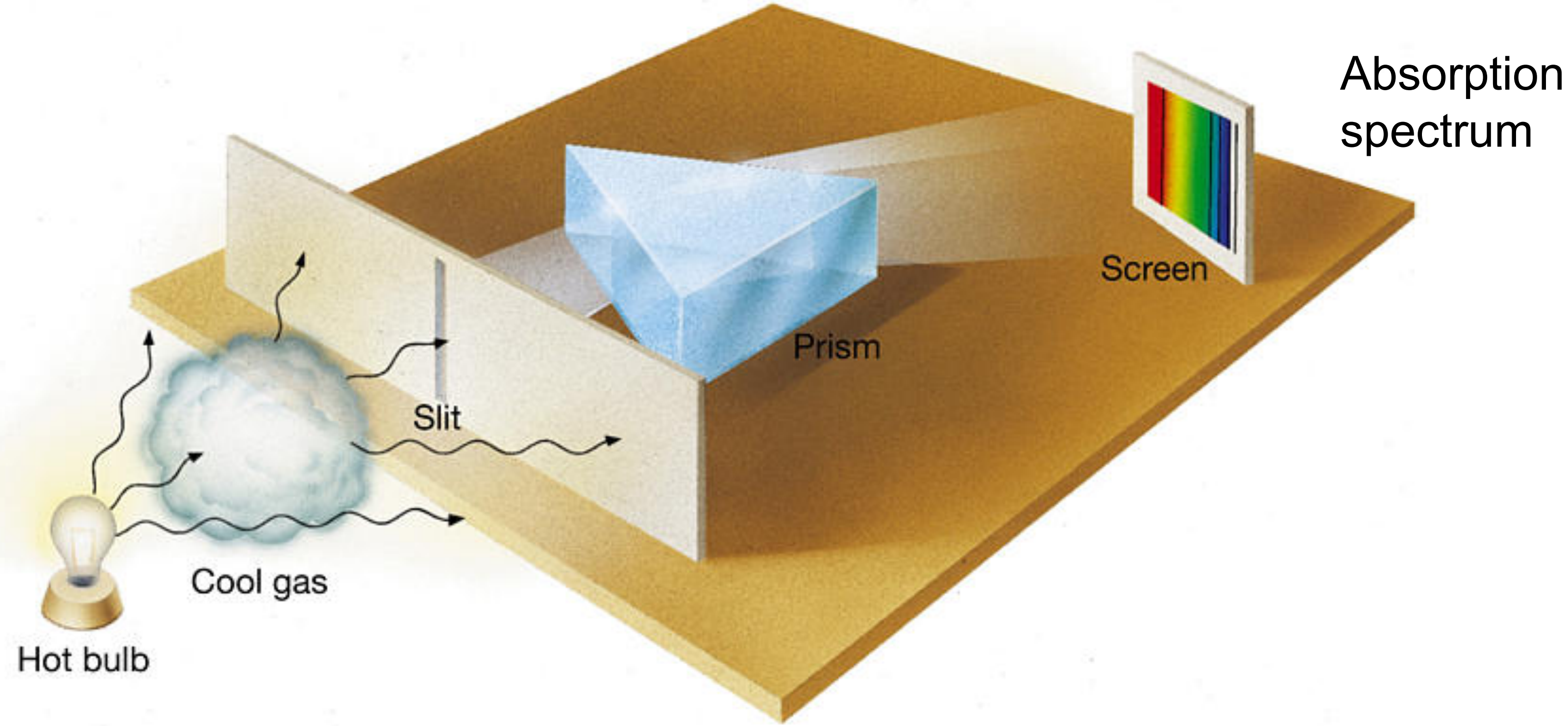


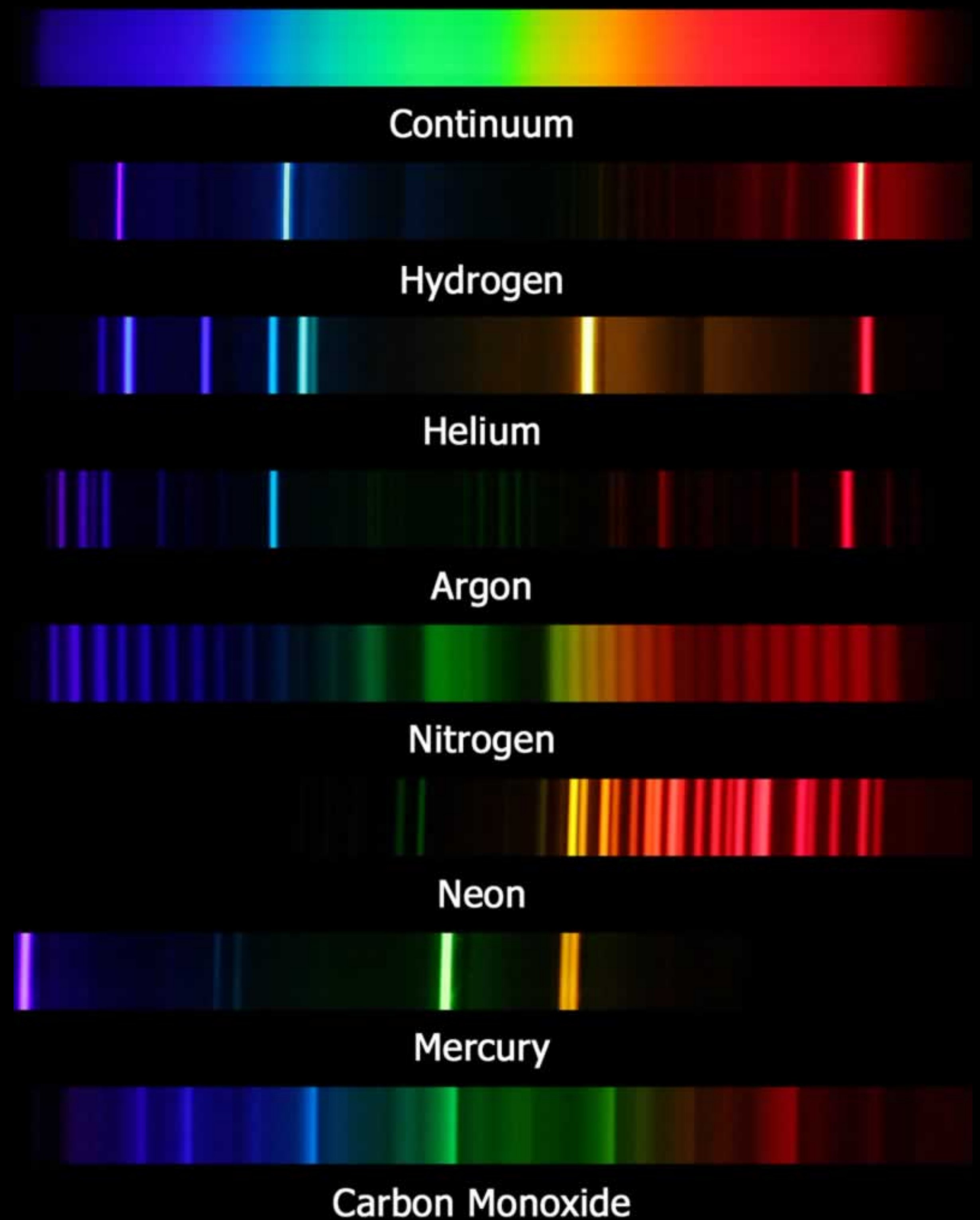

Continuous Spectrum
Emission Lines
Absorption Lines

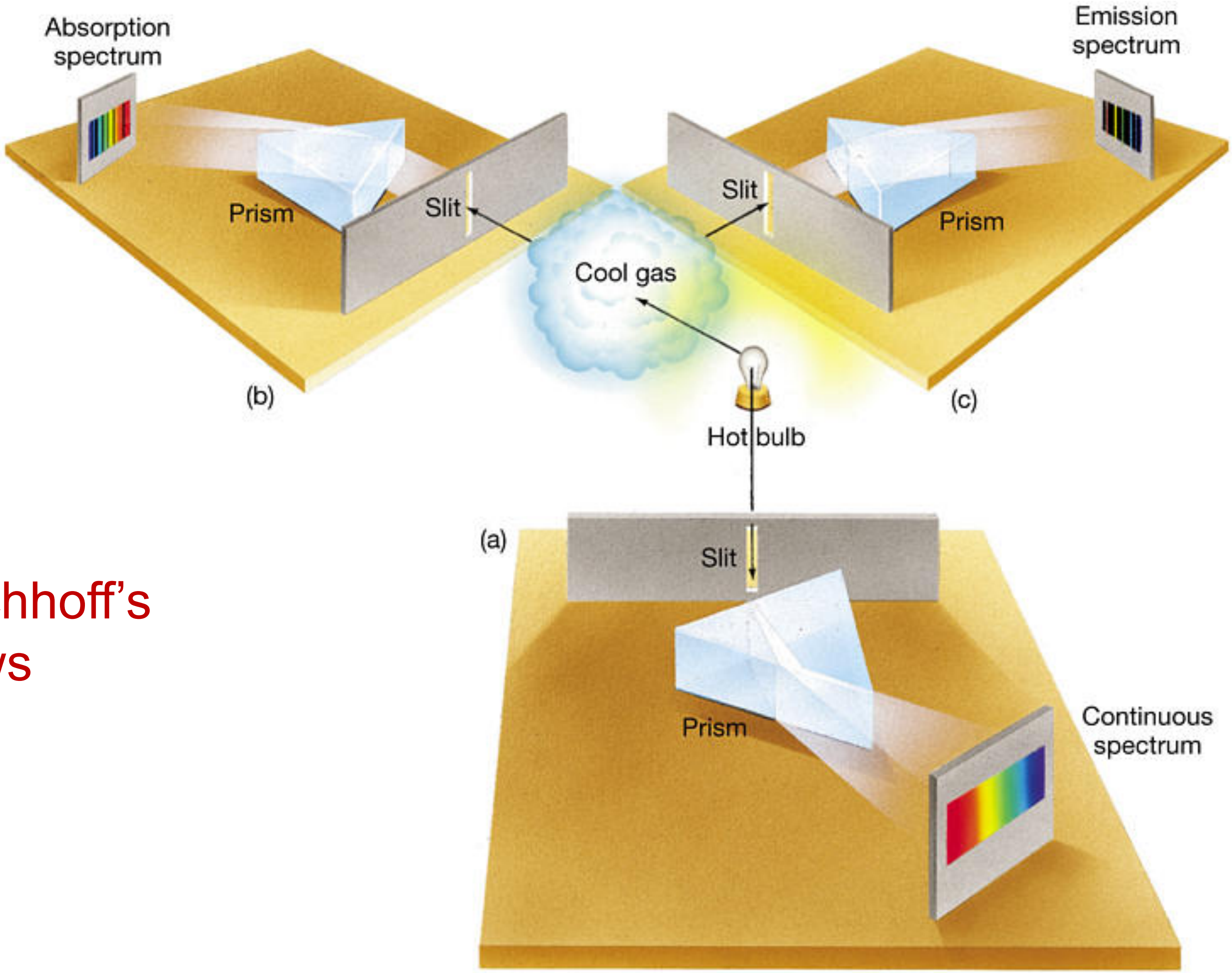


Continuous
spectrum

Emission
spectrum

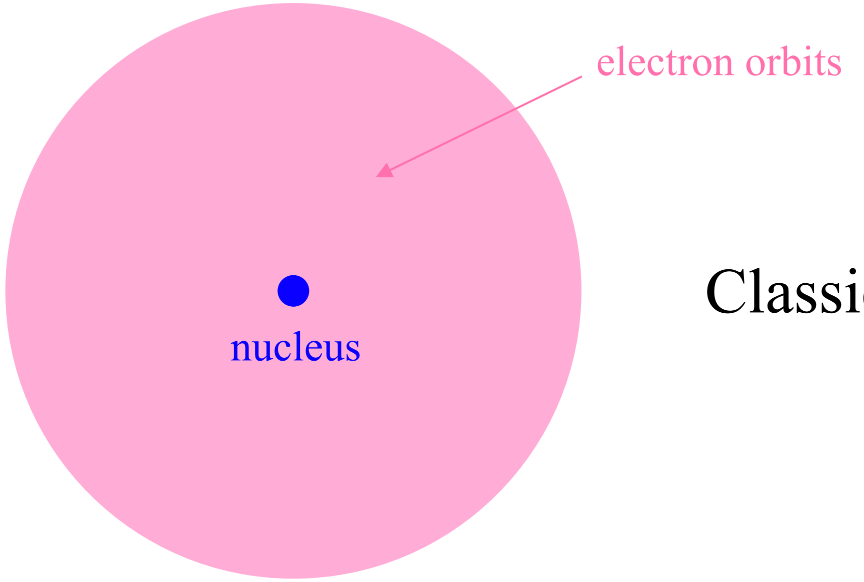

Absorption spectrum
Screen
Prism
Slit
Cool gas
Hot bulb


Continuum
Hydrogen
Helium
Argon
Nitrogen
Neon
Mercury
Carbon Monoxide

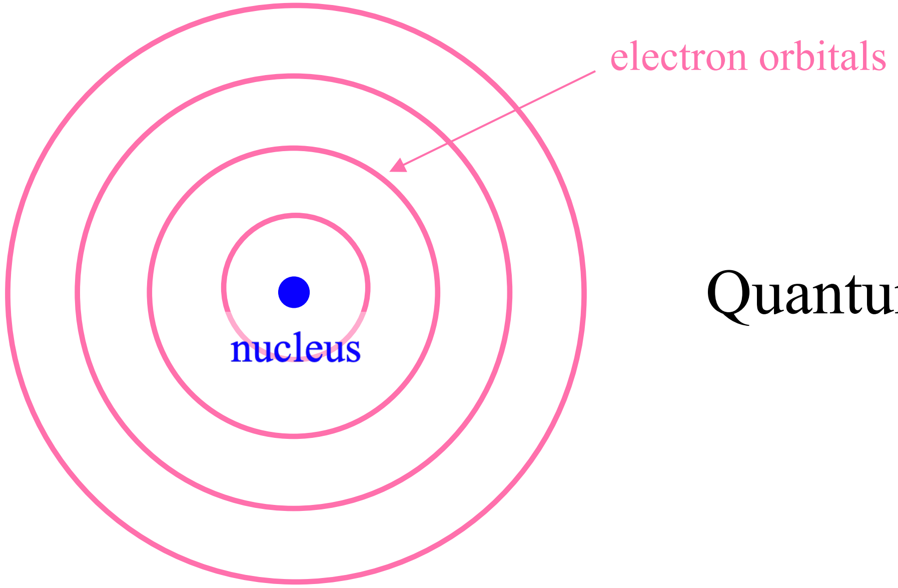


Kirchhoff's Laws

Quantization of Electron Energies

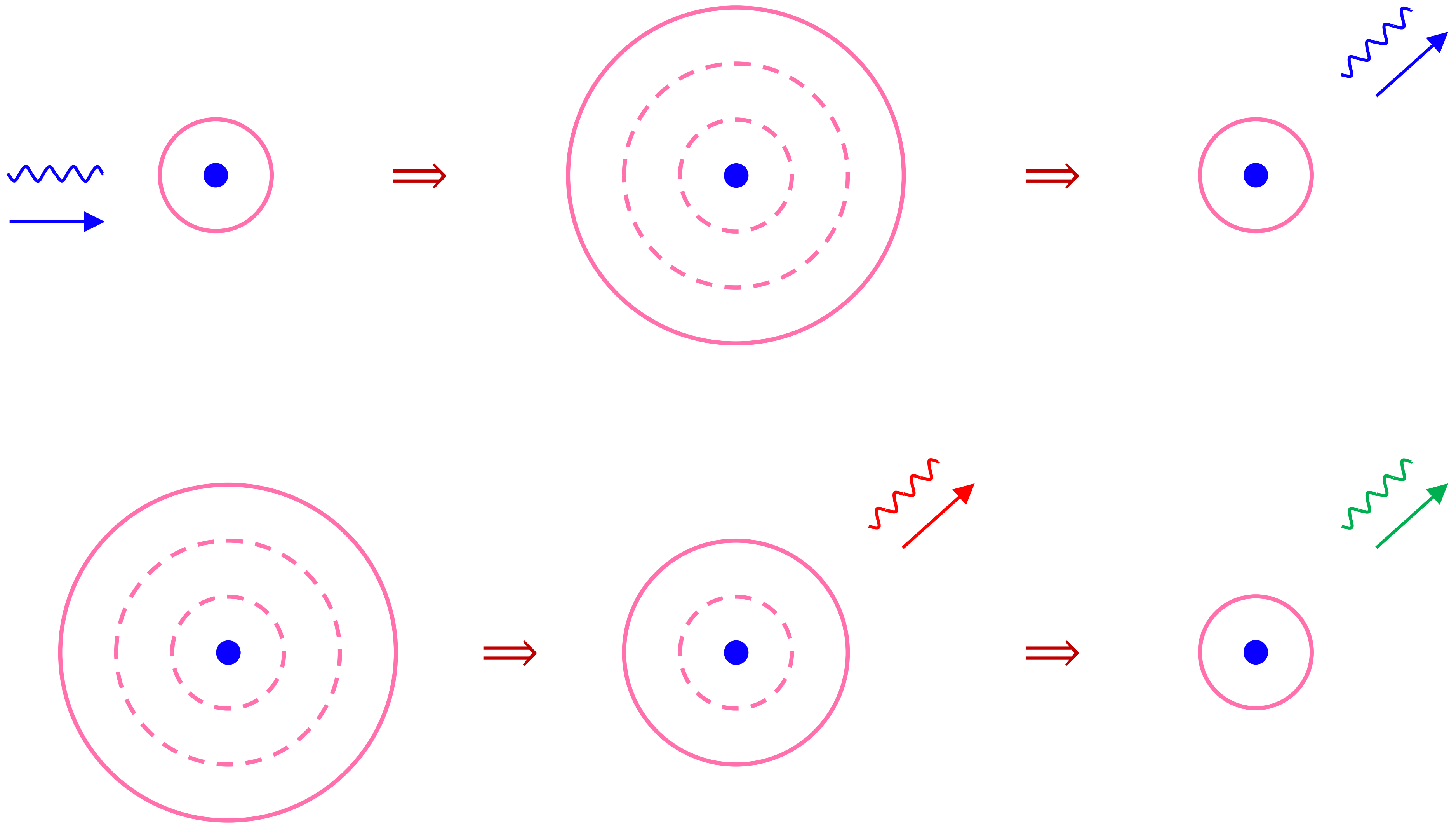


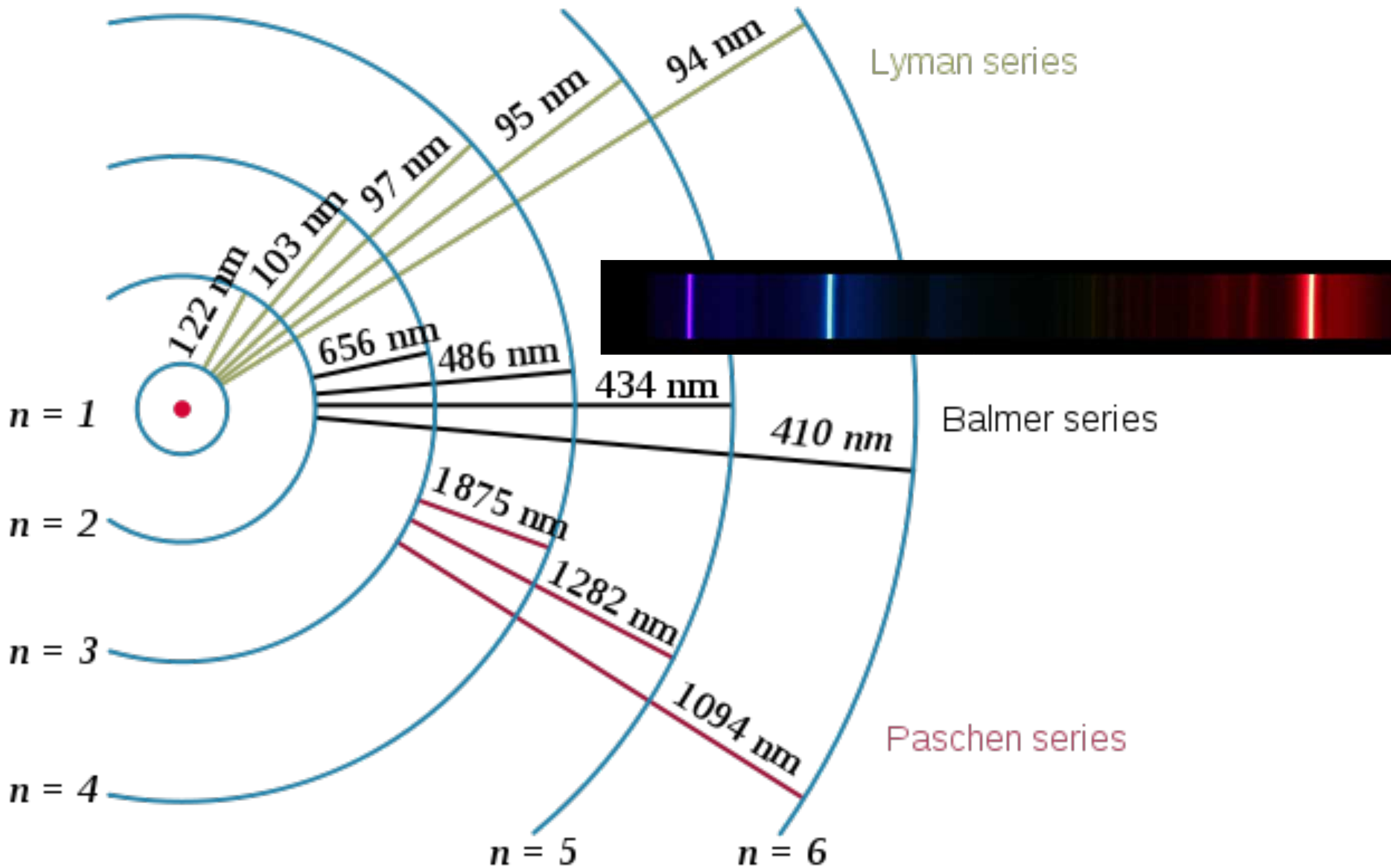
Classical picture: all electron orbits allowed



Quantum picture: only specific electron orbits allowed

Photon-Atom Interaction



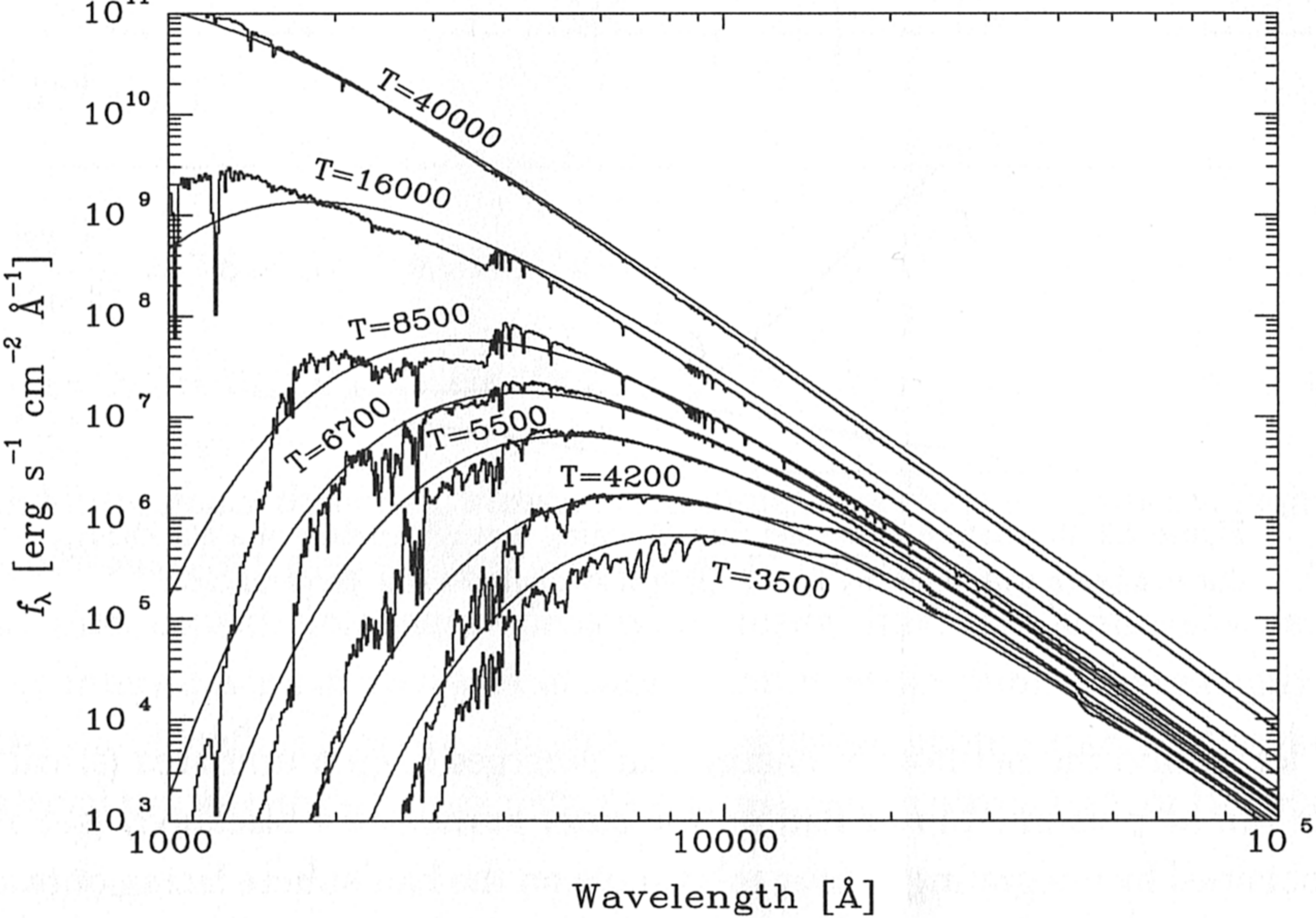


$$E_n = -13.6 \text{ eV} / n^2 \qquad 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

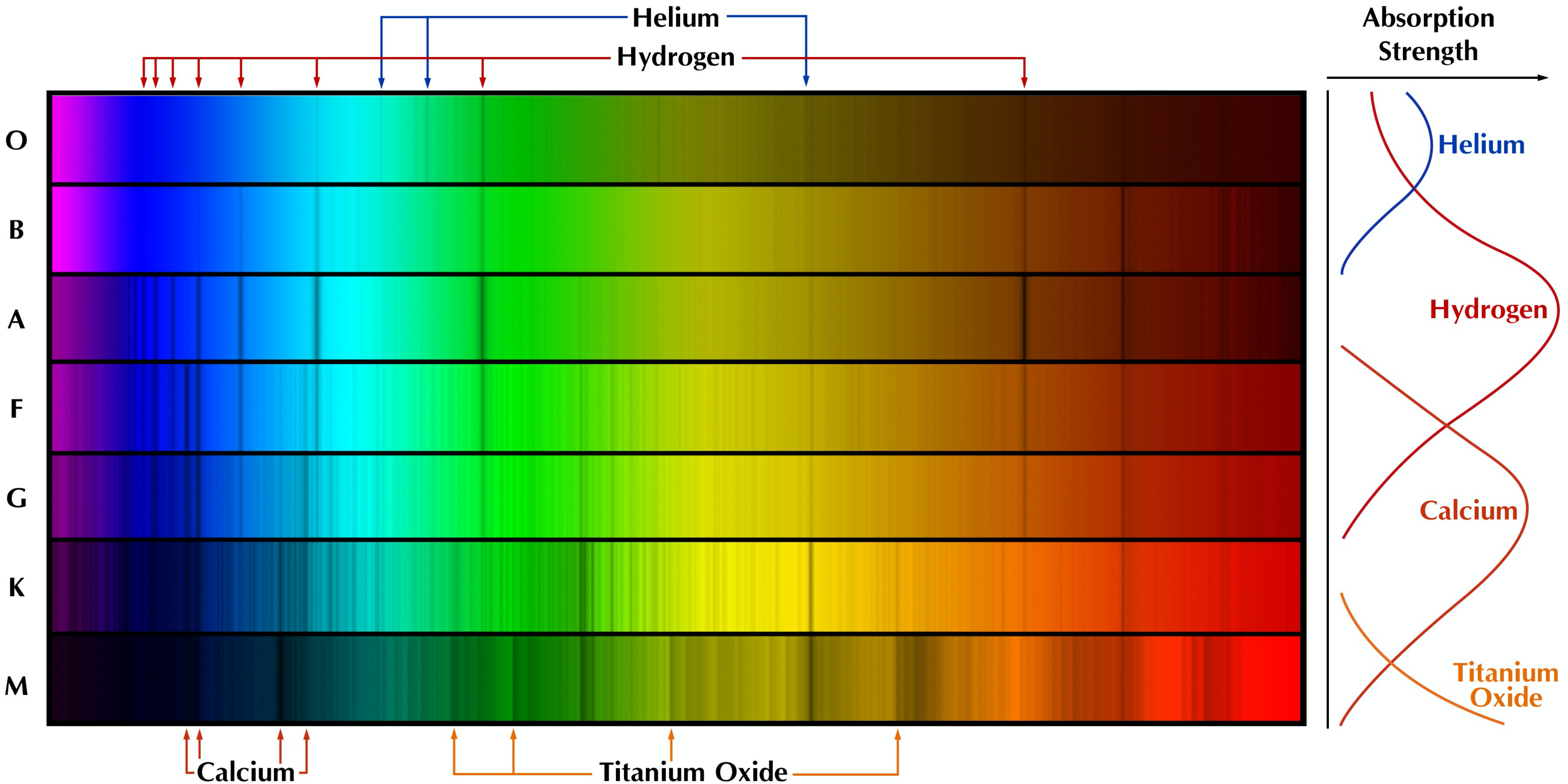
$$h\nu = En - E_m$$

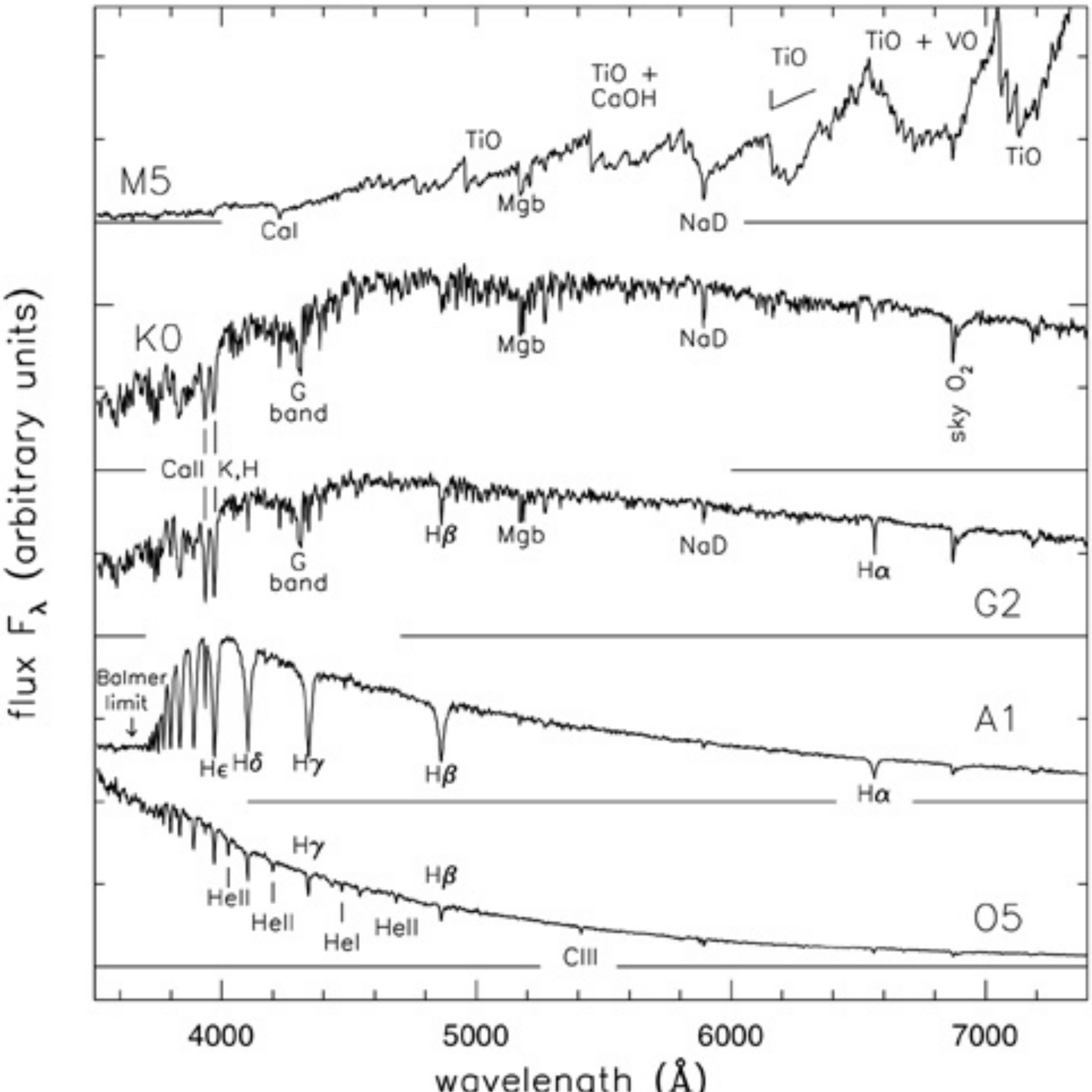
Solar Composition

Element	% by Volume	% by Mass
Hydrogen	90.946	70.682
Helium	8.913	27.509
Oxygen	0.077	0.954
Carbon	0.033	0.303
Neon	0.011	0.170
Nitrogen	0.010	0.108
Magnesium	0.004	0.068
Iron	0.003	0.137
Silicon	0.003	0.069



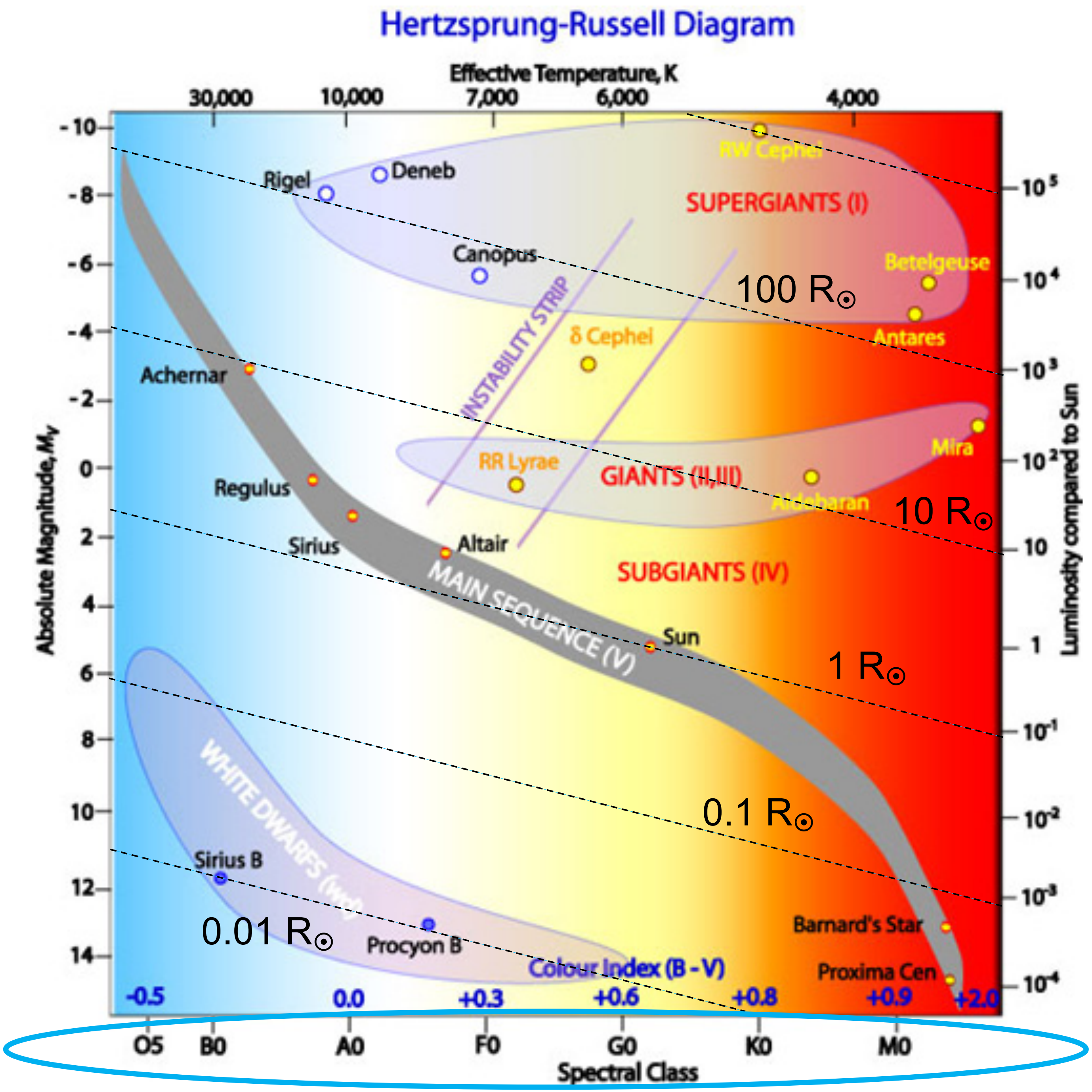
T=40000
T=16000
T=8500
T=6700
T=5500
T=4200
T=3500
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
1000
10000
10^{5}
f_λ [erg s^{-1} cm^{-2} Å^{-1}]
Wavelength [Å]


Helium
Hydrogen
O
B
A
F
G
K
M
Calcium
Titanium Oxide
Absorption
Strength
Helium
Hydrogen
Calcium
Titanium
Oxide



M5
TiO
TiO +
CaOH
TiO
TiO + VO
TiO
CaI
Mgb
NaD
K0
G
band
Mgb
NaD
sky O2
CaII K,H
G2
Hβ
Mgb
NaD
Hα
A1
Balmer
limit
Hϵ
Hδ
Hγ
Hβ
Hα
O5
Hγ
Hβ
HeII
HeII
HeI
HeII
CIII
flux Fλ (arbitrary units)
4000
5000
6000
7000
wavelength (Å)

Spectral Class	Intrinsic Color	Temperature (K)	Prominent Absorption Lines
O	Blue	41,000	He+, O++, N++, Si++, He, H
B	Blue	31,000	He, H, O+, C+, N+, Si+
A	Blue-white	9,500	H(strongest), Ca+, Mg+, Fe+
F	White	7,240	H(weaker), Ca+, ionized metals
G	Yellow-white	5,920	H(weaker), Ca+, ionized & neutral metal
K	Orange	5,300	Ca+(strongest), neutral metals strong, H(weak)
M	Red	3,850	Strong neutral atoms, TiO



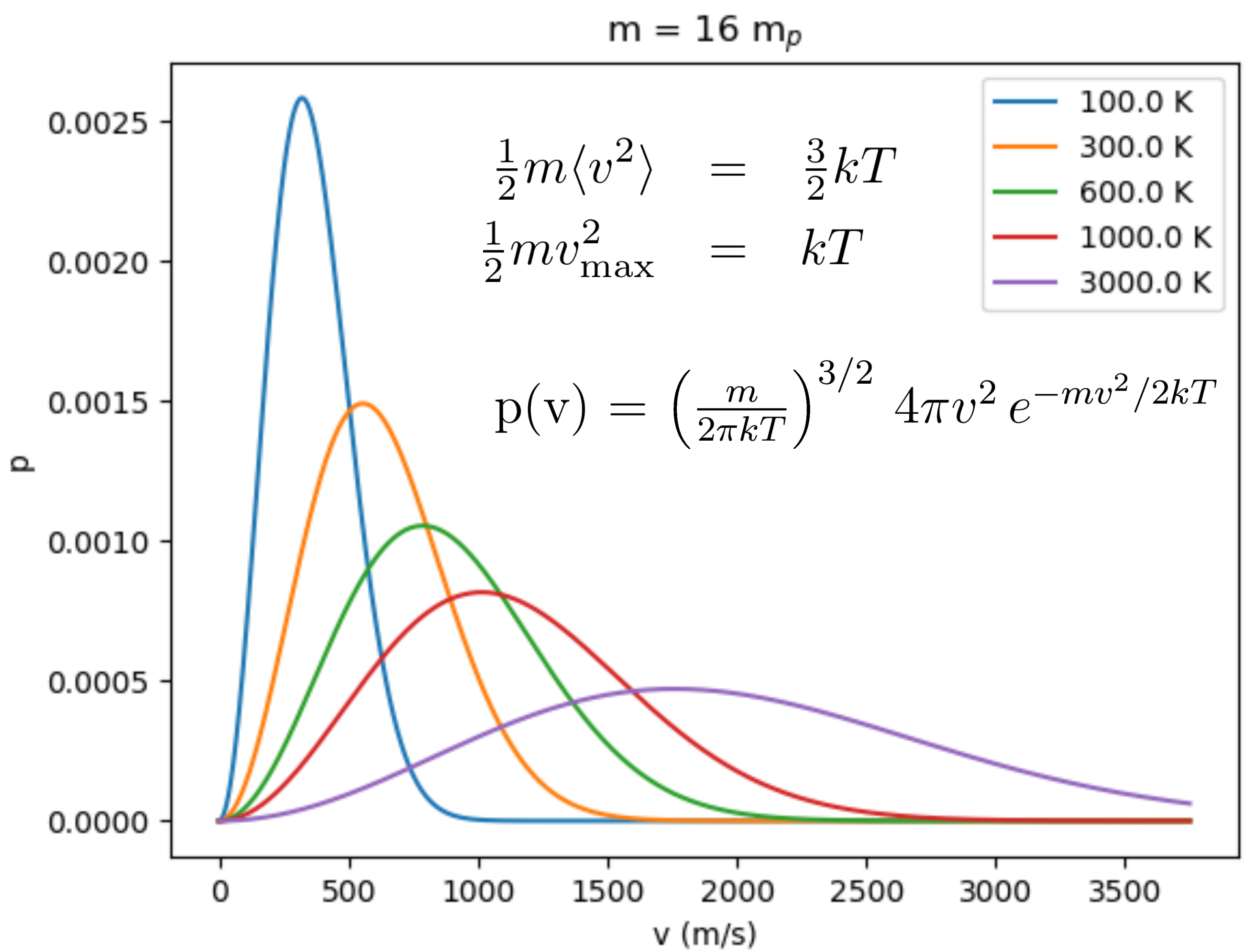
Hertzsprung-Russell Diagram
Effective Temperature, K
30,000
10,000
7,000
6,000
4,000
Absolute Magnitude, M_V
Luminosity compared to Sun
RW Cephei
Rigel
Deneb
SUPERGIANTS (I)
Canopus
Betelgeuse
100 R☉
Antares
INSTABILITY STRIP
δ Cephei
Achernar
Mira
RR Lyrae
GIANTS (II,III)
Regulus
Aldebaran
10 R☉
Sirius
Altair
SUBGIANTS (IV)
MAIN SEQUENCE (V)
Sun
1 R☉
WHITE DWARFS (wd)
0.1 R☉
Sirius B
Barnard's Star
0.01 R☉
Procyon B
Colour Index (B - V)
Proxima Cen
-0.5
0.0
+0.3
+0.6
+0.8
+0.9
+2.0
O5
B0
A0
F0
G0
K0
M0
Spectral Class

Maxwell-Boltzmann Distribution

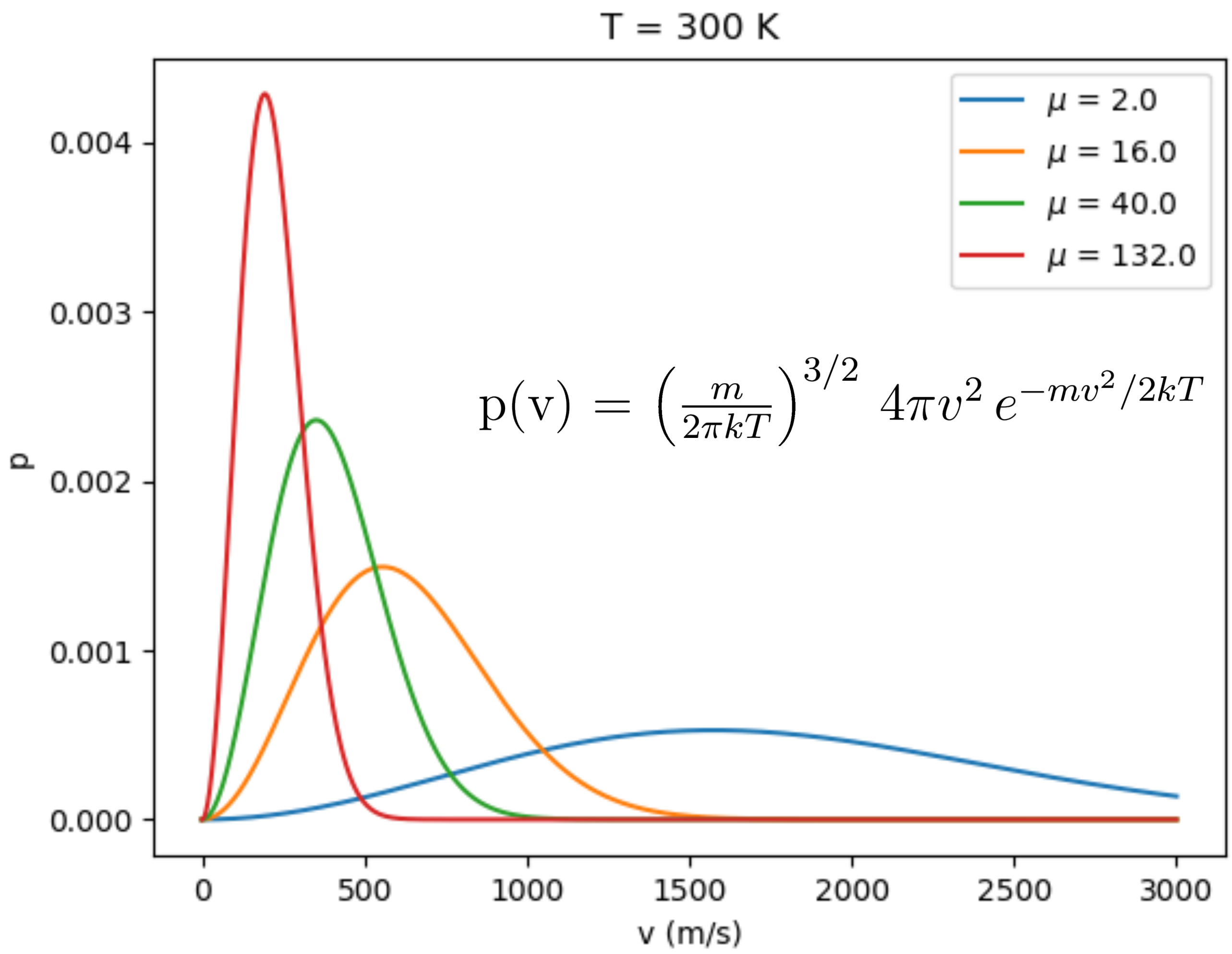
Most probable distribution of particle (mass m) velocities in thermal equilibrium at temperature T

$$\mathrm{p}(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \, e^{-mv^2/2kT}$$

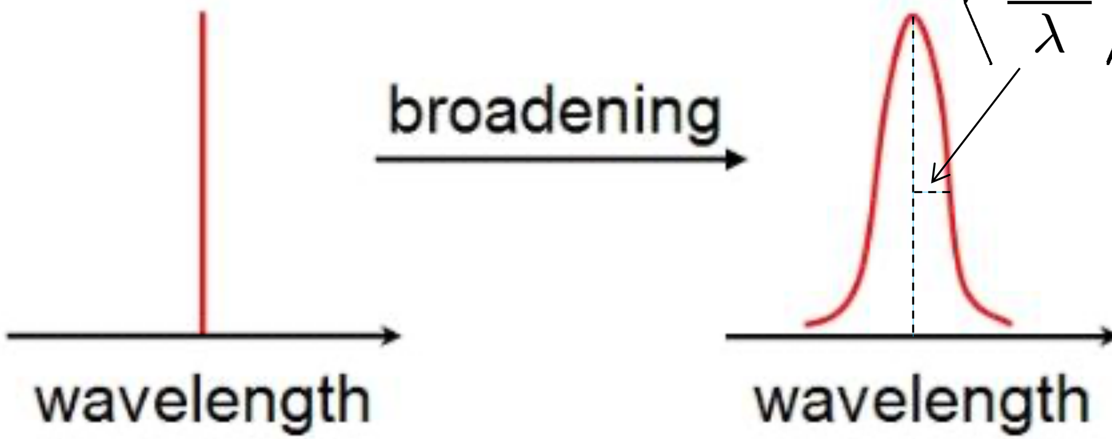
Maxwell-Boltzmann Distribution



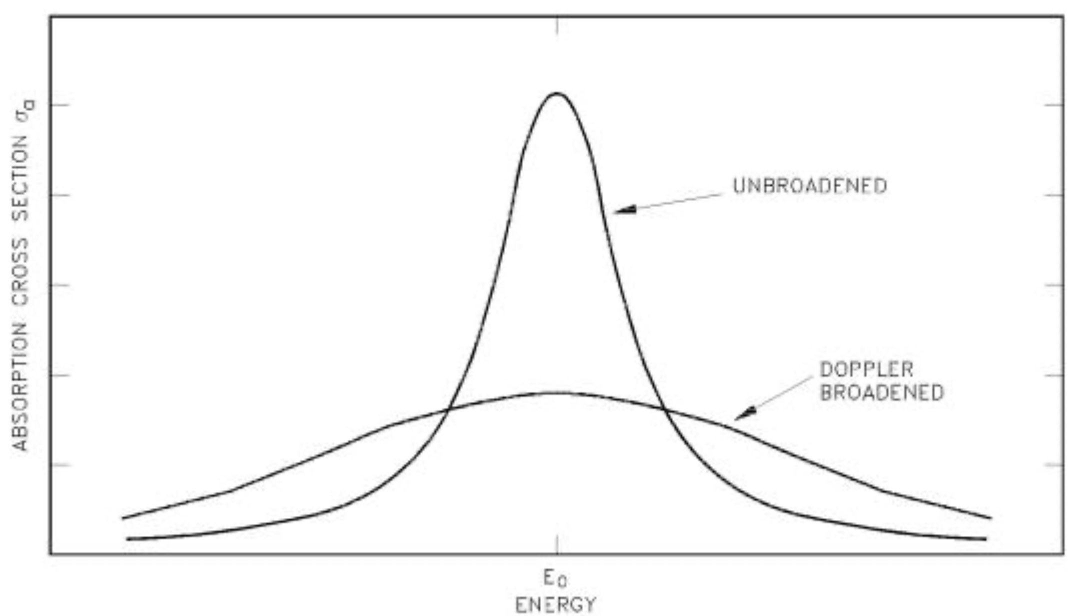
Maxwell-Boltzmann Distribution



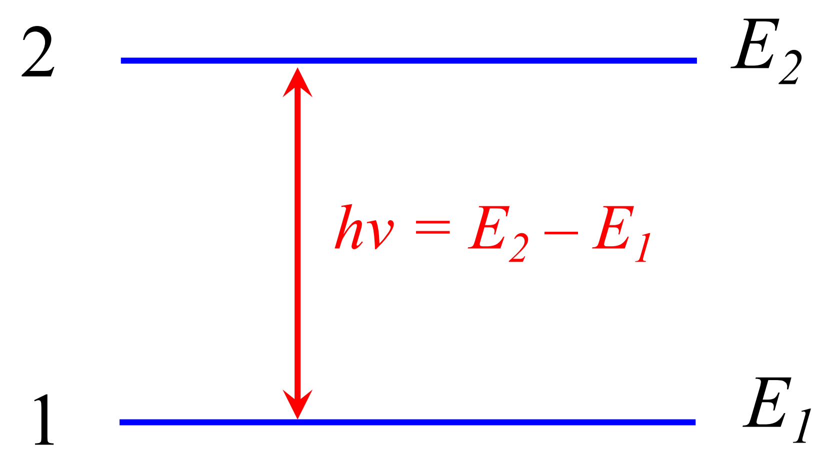
Line Broadening



$$\left\langle \frac{\Delta\lambda}{\lambda} \right\rangle \sim \sqrt{\frac{3kT}{mc^2}}$$



Boltzmann Equation



$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT}$$

Hydrogen: $g_n = n^2$

Saha Equation

$$[i] \longleftrightarrow [i+1] + e^-$$

excitation potential

$$\frac{n_{i+1} n_e}{n_i} = \frac{2}{\Lambda^3} \frac{g_{i+1}}{g_i} e^{-\chi_i/kT}$$

$$\Lambda = \sqrt{\frac{h^2}{2\pi m_e kT}}$$

Hydrogen ionization

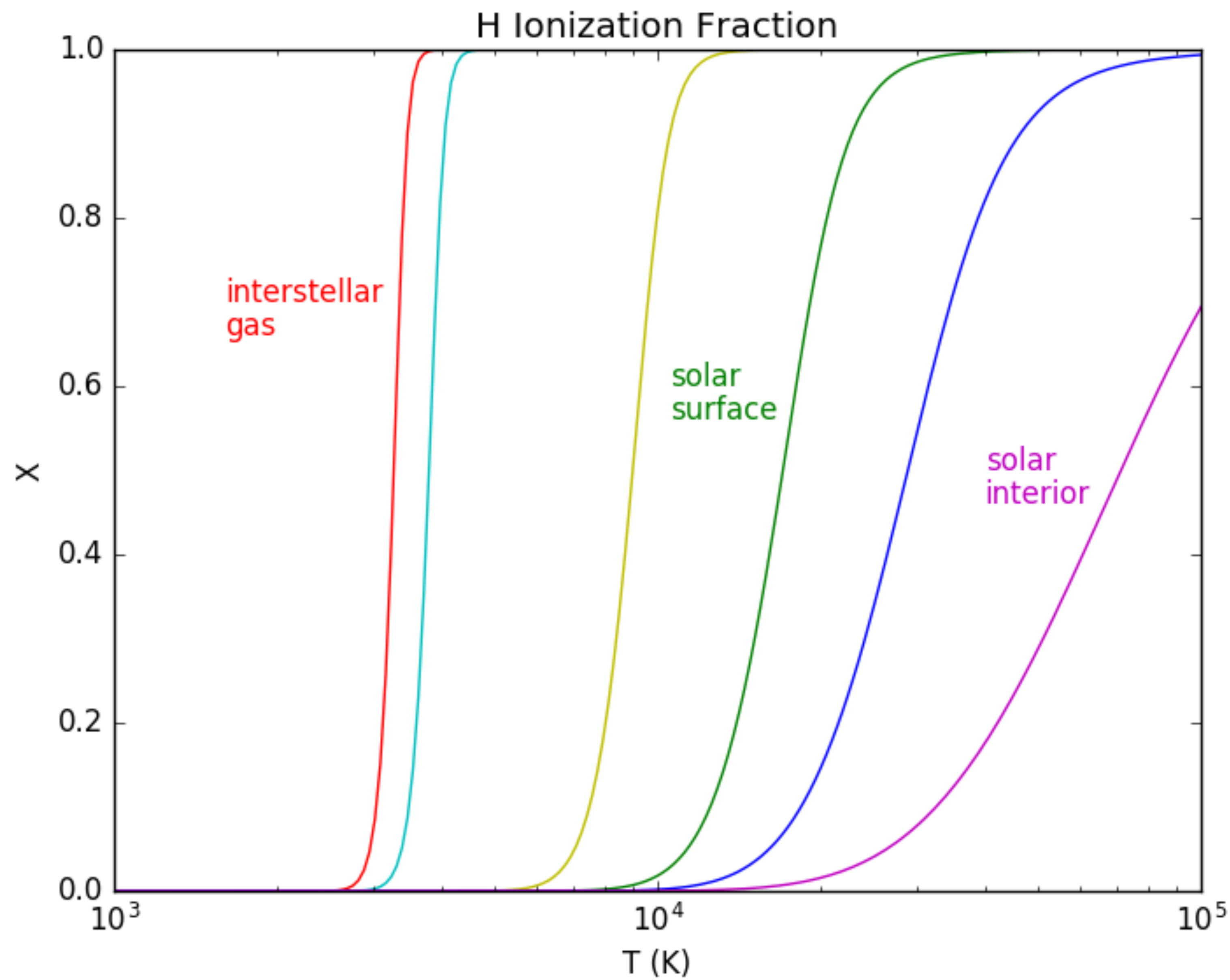
$$n_H \qquad\qquad n_p \; = \; n_e$$

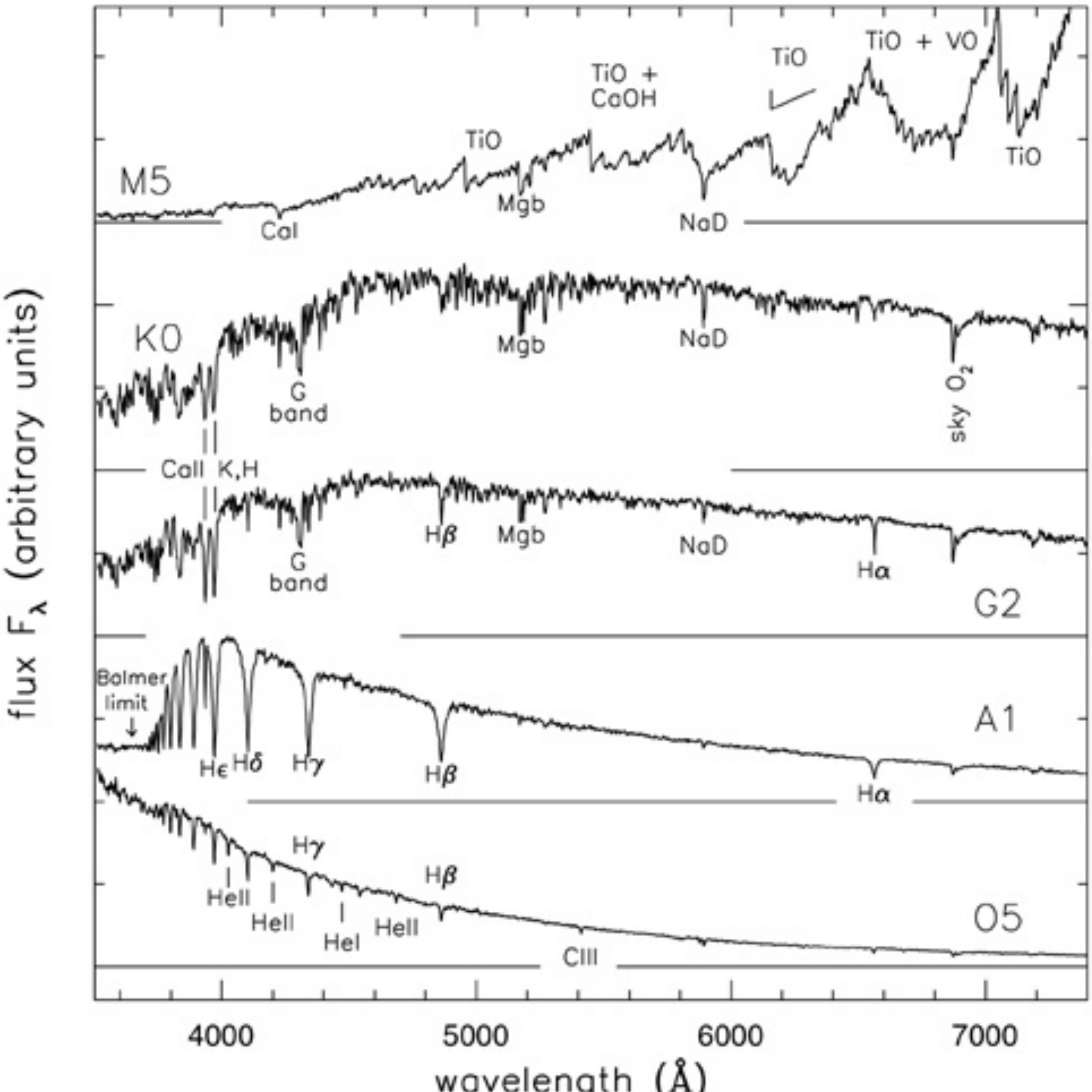
$$\mathrm{H} \longleftrightarrow \mathrm{p} + e^-$$

χ = 13.6 eV

$$\frac{X^2}{1-X} = \frac{1}{n_H \Lambda^3} e^{-\chi/kT}$$

$$X = \frac{n_e}{n_H}$$


H Ionization Fraction
1.0
0.8
0.6
0.4
0.2
0.0
X
10^3
10^4
10^5
T (K)
interstellar gas
solar surface
solar interior


M5
TiO
TiO + CaOH
TiO
TiO + VO
TiO
CaI
Mgb
NaD
K0
Mgb
NaD
G band
sky O_2
CaII K,H
G2
Hβ
Mgb
NaD
Hα
G band
A1
Balmer limit
Hε
Hδ
Hγ
Hβ
Hα
O5
Hγ
Hβ
HeII
HeII
HeI
HeII
CIII
flux F_λ (arbitrary units)
4000
5000
6000
7000
wavelength (Å)

Spectral Class	Intrinsic Color	Temperature (K)	Prominent Absorption Lines
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A	Blue-white	9,500	H(strongest), Ca+, Mg+, Fe+
F	White	7,240	H(weaker), Ca+, ionized metals
G	Yellow-white	5,920	H(weaker), Ca+, ionized & neutral metal
K	Orange	5,300	Ca+(strongest), neutral metals strong, H(weak)
M	Red	3,850	Strong neutral atoms, TiO