# PHYS 231: Introductory Astrophysics

*Winter 2020*

## Homework #6

(Due: 5 pm, March 13, 2020)

*Each problem is worth 20 points. Answer 7 problems. If you wish, you may answer all 8 for extra credit.*

1. A spinning neutron star of mass $M = 1.3\,M_\odot$ has uniform density, radius $R = 10$ km, and rotation period $P = 1$ s. It is accreting from a companion at a rate $\dot{M} = 3 \times 10^{-9} M_\odot\,\mathrm{yr}^{-1}$. Assume that the accreted material is moving in a circular Keplerian orbit around the neutron star just before it hits the surface, and that all of its angular momentum is transferred to the neutron star.

   (a) What are the orbital speed and angular momentum per unit mass of the accreted material just before it reaches the neutron star? Hence write down an expression for the rate at which the neutron star's angular momentum is changing.

   (b) The angular momentum of the neutron star is $I\Omega$, where $\Omega$ is its angular speed and $I = \frac{2}{5}MR^2$ is its moment of inertia (assumed constant). By differentiating this expression and equating it to the answer to part (a), derive a differential equation for $\dot{\Omega}$, the rate of increase of the neutron star's angular speed.

   (c) Neglecting changes in $M$ and $R$, solve this equation and calculate how long it will take for the neutron star to reach the theoretical minimum rotation period of 1 millisecond.

2. What are the Jeans length and Jeans mass of

   (a) a molecular cloud core of density $10^6$ $H_2$ molecules per cubic centimeter and temperature 20 K

   (a) a cool HI cloud of density $30\,H$ atoms per cubic centimeter and temperature 250 K?

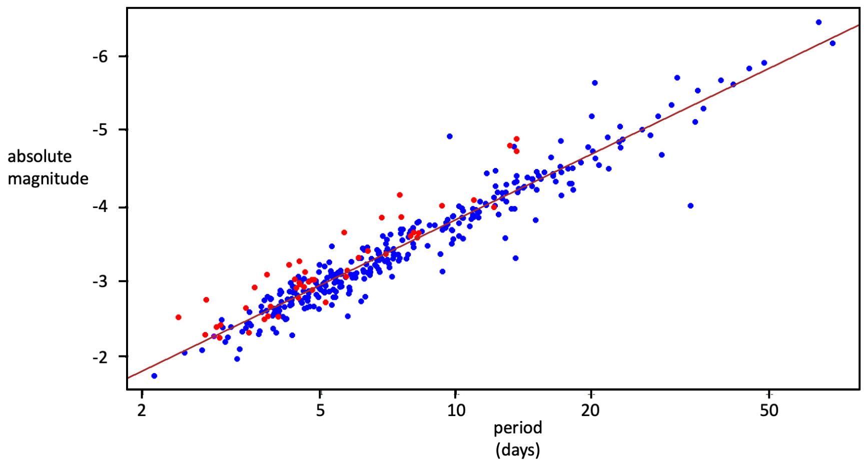
   (a) a region of warm ionized medium with density 0.1 hydrogen masses per cubic centimeter and temperature $10^4$ K?

   In each case, assume composition $X = 0.7, Y = 0.3$.

3. Consider a system consisting of a body of mass $m$ on a circular orbit of radius $R$ around a larger body of mass $M \gg m$. Assume that the mass $M$ is at the center of mass of the system and the angular speed of the smaller mass is $\Omega = \sqrt{GM/R^3}$.

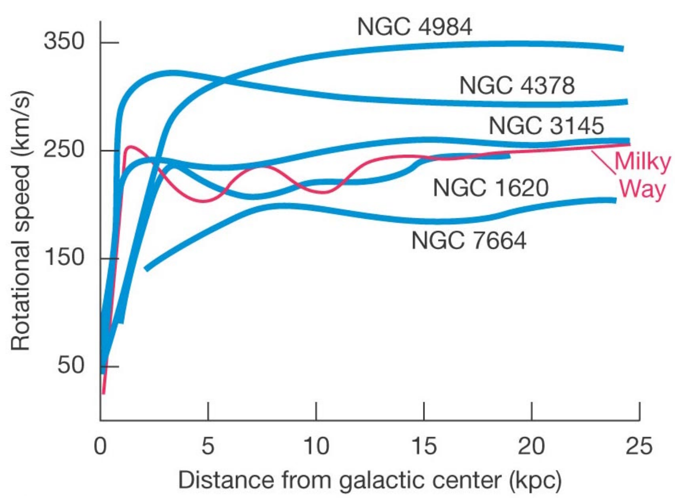
   Now consider the motion of a test particle in the frame of reference rotating with the orbit of mass $m$. For simplicity, we'll look only at the case where the test particle lies somewhere along the line joining the two masses. Choose a coordinate system $x$ along that line, where the mass $M$ is at $x = 0$ and mass $m$ is at $x = R$. In the rotating frame, the test particle feels the gravitational acceleration due to the two masses, as well as the centrifugal acceleration due to the rotating frame, $\Omega^2 x$. Find the locations of the three Lagrange points along the line, where the total acceleration of the test particle is zero. (Note: one of them will have $x < 0$.)

4. (a) A Cepheid variable star in a distant galaxy obeys the period–luminosity relation shown in the accompanying figure (see also Fig. 8.1 in the textbook). If its period is measured to be 20 days and its apparent magnitude is 19, calculate the distance to the galaxy.




(b) Type Ia supernovae are found to have remarkably consistent absolute magnitudes of $M = -19.6$, with very little scatter. The Hubble Space Telescope has an effective limiting magnitude of about 27 in the relevant wavelength range. What is the maximum distance at which it could just detect a Type Ia supernova?

5. (a) For each of the 6 galaxies whose rotation curves are shown below, estimate the total mass contained within 20 kpc from the galactic center.




(b) For a spherically symmetric distribution of mass, a flat rotation curve, $V(R) = V_c$, corresponds to a density distribution $\rho(r) = Ar^{-2}$. By integrating the density from $r = 0$ to $r = R$ to obtain the mass $M(R)$, calculate $A$ in terms of $V_c$.

6. As discussed in class, the Eddington luminosity, $L_{edd}$, is the luminosity of a source at which the radiation force on accreting material exactly balances the force of gravity. For completely ionized pure hydrogen, we saw that

$$L_{edd} = \frac{4\pi c G M m_p}{\sigma_T},$$

where $M$ is the mass of the source, $m_p$ is the mass of a proton, and $\sigma_T$ is the Thomson cross section. This luminosity represents an effective maximum luminosity for an accreting source. For luminosities greater than the Eddington value, radiation prevents matter from falling onto the source, shutting down the accretion flow. (It's not a completely realistic limit, since accreting sources don't accrete spherically, but it sets a useful scale for discussion.)

(a) Evaluate the Eddington luminosity for accretion onto a $10^9\ M_\odot$ black hole. How does it compare to the luminosity $10^{41}$ W of a bright AGN?

(b) Accretion in a disk onto a black hole is a very efficient energy production mechanism, converting 5.7% of the infalling rest mass into radiation. Assuming Eddington luminosity, calculate the rate at which a $10^9\ M_\odot$ black hole accretes mass.

(c) A black hole of mass $M_0 = 1000 M_\odot$ at time $t = 0$ starts accreting at the Eddington rate. It continues to accrete at the instantaneous Eddington rate as its mass increases. How long does it take for its mass to reach $2 \times 10^9 M_\odot$?

7. A cluster of galaxies is located 500 Mpc from Earth. You are able to take measurements on 10 of the galaxies it contains. The distances, in arc seconds, from the cluster center, and the measured redshifts of the ten galaxies are given by

| x ($''$) | y ($''$) | $z$ (redshift) |
|---|---|---|
| -368 | 161 | 0.1185 |
| -208 | -350 | 0.1213 |
| -219 | 637 | 0.1191 |
| -276 | -388 | 0.1200 |
| 397 | 462 | 0.1192 |
| 289 | 587 | 0.1196 |
| -33 | -517 | 0.1189 |
| -547 | 37 | 0.1183 |
| -123 | 236 | 0.1205 |
| 163 | -246 | 0.1204 |

(a) What is the average redshift of the galaxies?

(b) What is the standard deviation of the redshifts?

(c) What is the 1-D line of sight velocity dispersion $\sigma$ of the galaxies? (Hint: Just convert the redshift dispersion found in part (b) converted to velocity by using of the Doppler relation.)

(d) What is the radius $R$ of the cluster (in Mpc), defined as the maximum distance of any galaxy from the center.

(e) Use the virial theorem, $U = -2K$, $K \approx \frac{3}{2} M\sigma^2$, to estimate the total mass $M$ of the cluster in solar masses. You may assume that the galaxies are distributed in a uniform sphere, which means that the total potential energy is

$$U = -\tfrac{3}{5}\frac{GM^2}{R}.$$

(f) Use the given distance and mean redshift of the ten galaxies to estimate the Hubble constant, $H_0$.

8. A galaxy is observed at a redshift of z = 0.1, in a universe with a Hubble constant of $H_0 = 70$ km/s/Mpc.

   (a) How far away is the galaxy?

   (b) If the galaxy has an effective radius of 15 kpc, what is its angular size on the sky?

   (c) If the galaxy has a peculiar velocity of 600 km/s, what redshift would that correspond to? What fractional error would it introduce into the cosmological redshift?