PHYS 231: Introductory Astrophysics

Winter 2020

Homework #3

(Due: January 29, 2020)

Each problem is worth 20 points.

1. In a fully ionized gas, the dominant contribution to the opacity of a star is *Thomson scattering* of photons on electrons, defined by $\alpha = n_e \sigma_T$, where n_e is the number density of electrons and σ_T is the Thomson cross-section, $\sigma_T = 6.7 \times 10^{-29}$ m^2. If X, Y, and Z are the fractions by mass of hydrogen (mass $= m_H = 1.67 \times 10^{-27}$ kg, 1 electron), helium (mass $= 4m_H$, 2 electrons), and "metals" (half an electron per m_H), respectively, calculate the electron density n_e, the mean free path of a photon ℓ, and the mean mass per particle $\bar{m}$ in the following environments: (i) just below the solar photosphere, where $X = 0.75$, $Y = 0.24$, $Z = 0.01$, and $\rho = 10$ kg m^{-3}, (ii) in the solar interior, where $X = 0.71$, $Y = 0.27$, $Z = 0.02$, and $\rho = 10^3$ kg m^{-3}, and (iii) in the solar core, where $X = 0.34$, $Y = 0.64$, $Z = 0.02$, and $\rho = 10^5$ kg m^{-3}.

2. The intensity I of a beam of light traveling through a uniform medium varies with distance x along the direction of the beam according to the *transfer equation*

$$\frac{dI}{dx} = -\alpha I + \epsilon,$$

where α and ϵ are constants. The intensity of the beam at $x = 0$ is I_0.

(a) Show that, by writing $y = I - \epsilon/\alpha$, this equation may be written in the form

$$\frac{dy}{dx} = -\alpha y.$$

(b) Hence write down a solution to the original equation, and show that the intensity I effectively "forgets" its initial value I_0 for $x \gg 1/\alpha$.

3. Repeat the argument starting at Equation 3.22 of Maoz, for a relativistic gas with $PV = \frac{1}{3}E_{th}$. What is the resulting virial relation, and what can you say about the total energy of the star?

4. The gas and radiation pressures in a (nonrelativistic) star of density ρ and temperature T are given by

$$P_{gas} = \frac{\rho k T}{\bar{m}}, \qquad P_{rad} = \tfrac{1}{3} a T^4.$$

Calculate the ratio P_{rad}/P_{gas} for the same gas composition and at the same three environments as in Problem 1: (i) $\rho = 10$ kg m^{-3}, $T = 10^5$ K, (ii) $\rho = 10^3$ kg m^{-3}, $T = 10^6$ K, and (iii) $\rho = 10^5$ kg m^{-3}, $T = 10^7$ K.