PHYS 231: Introductory Astrophysics

Winter 2020

Homework #2

(Due: January 22, 2020)

Each problem is worth 20 points.

1. (a) Show that, if the ratio of blackbody fluxes f_ν at two different frequencies (ν_1 and ν_2, say) is known, then we can in principle determine the temperature T.

   (b) Explain why this is difficult to accomplish in practice if the measurements are made in the Rayleigh-Jeans part of the spectrum, with $h\nu \ll kT$.

   (c) If both measurements are in the Wien tail, with $h\nu \gg kT$, write down a simple approximate expression for the temperature in terms of the flux ratio $f = f_\nu(\nu_1)/f_\nu(\nu_2)$.

2. Neutral helium has an emission line at 389 nm.

   (a) What is the frequency of the emitted photon?

   (b) What is the energy of that transition in electron volts (eV)?

   (c) Imagine that a helium cloud was traveling away from you at 2000 km/s when this transition took place. At what wavelength would you observe the emitted photon?

3. The figures below show (i) the observed color–magnitude diagram for the Hyades star cluster, and (ii) a theoretical diagram (with absolute, rather than apparent magnitudes on the y-axis, note) for a model cluster of stars a few hundred million years old.

   (a) Choose a few "typical" main sequence stars, at $B - V = 0.25, 0.5, 1,$ and 1.5. For each color, what the approximate apparent magnitude of such a star in the Hyades cluster?

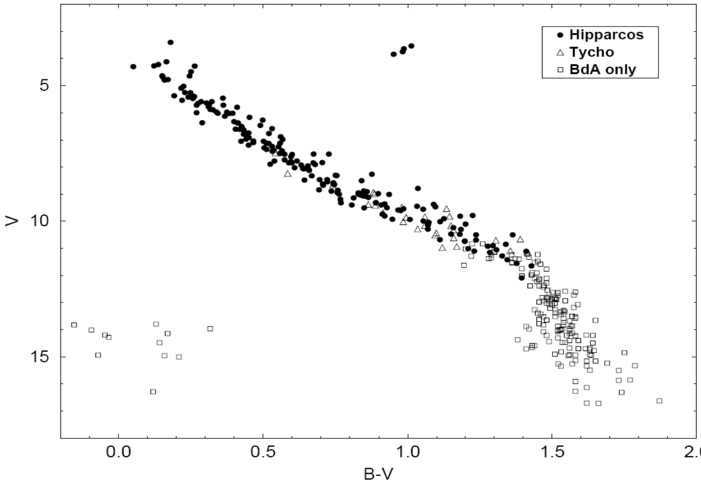
   (b) For the same colors, what are the absolute magnitudes in the theoretical model?

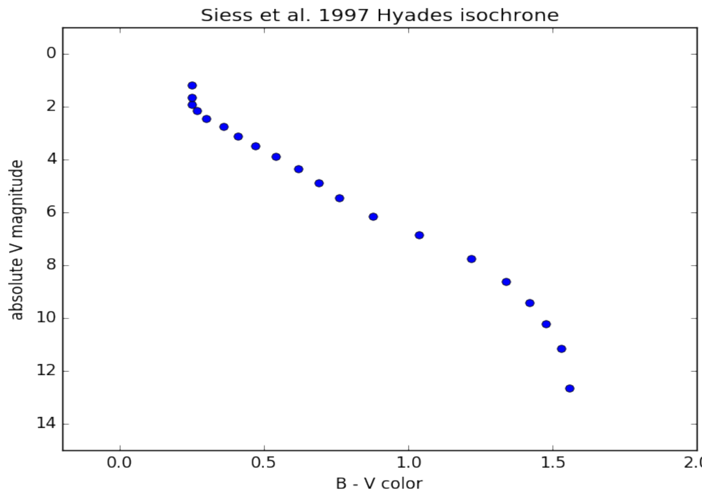
   (c) Hence, for each color, determine the distance modulus, and thus a distance estimate to the Hyades. What is the average of your estimates?

   (d) The Sun has an absolute magnitude of 4.8. Based on that, what is the luminosity, in solar units, of the brightest star still on the main sequence in the Hyades cluster?

   (e) Using the approximate relation $L \propto M^4$, estimate the mass of the most massive star in the Hyades still on the main sequence.

   (f) The lifetime of the Sun is approximately 10 billion years. Given the relation derived in class, $t_{MS} \propto M^{-3}$, estimate the age of the Hyades cluster.





4. Suppose a cloud of neutral atomic hydrogen is held at a density comparable to that of Earth's atmosphere, $n = 5 \times 10^{26}\ \mathrm{m}^{-3}$.

   Do each of the following calculations at a temperature of (i) 10 K (the temperature—but not the density—of molecular clouds in the interstellar medium); (ii) 6000 K (the temperature at the surface of the Sun); (iii) 1.5×10^7 K (a temperature typical of the center of a star). For each temperature, calculate

   (a) The characteristic energy kT (expressed in eV) of particles in the system.

   (b) The typical (rms) speed of an atom or nucleus.

   (c) The ratio of atoms in the $n = 4$ electronic state compared to the ground state $n = 1$. (Remember $g_n = 2n^2$.)

   (d) The ionization fraction, that is, the ratio

$$X = \frac{n_p}{n_H + n_p},$$

   where n_H is the number density of hydrogen atoms, n_p is the number density of ionized hydrogen nuclei (i.e. protons), and $n = n_H + n_p$ is constant.