PHYS 231: Introductory Astrophysics

Winter 2020

Homework #2 solutions

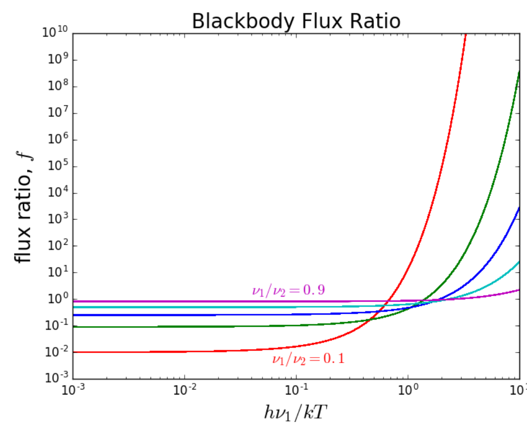
1. (a) The ratio of fluxes at frequencies ν_1 and ν_2 is

$$f = \frac{f_1}{f_2} = \frac{\nu_1^3}{\nu_2^3}\left(\frac{e^{h\nu_2/kT}-1}{e^{h\nu_1/kT}-1}\right).$$

Let's clean up the notation by defining $n = \nu_1/\nu_2$, assumed less than 1, and $x = h\nu_1/kT$. Then

$$f(x) = n^3\left(\frac{e^{x/n}-1}{e^x-1}\right).$$

The figure at right plots $f(x)$ for various values of n. Note that, in all cases, f is single valued, so any allowed value of f leads to a unique solution for x and hence T.



(b) For $h\nu \ll kT$, $f(x)$ is almost independent of x, so as a practical matter a small error in f leads to a large change in T, making it hard to measure the temperature accurately. We can see this explicitly by expanding the original definition of f for small ν_1 and ν_2:

$$f \approx \frac{\nu_1^3}{\nu_2^3}\left(\frac{h\nu_2/kT}{h\nu_1/kT}\right) = \left(\frac{\nu_1}{\nu_2}\right)^2,$$

which is independent of T.

(c) In the Wien tail, with $\nu_1, \nu_2 \gg kT$, we have

$$f \approx \frac{\nu_1^3}{\nu_2^3}\left(\frac{e^{h\nu_2/kT}}{e^{h\nu_1/kT}}\right) = \frac{\nu_1^3}{\nu_2^3}e^{h(\nu_2-\nu_1)/kT},$$

so

$$T = \frac{h(\nu_2-\nu_1)}{k\log_e\left(\frac{\nu_2^3 f}{\nu_1^3}\right)}.$$

2. An emission line at 389 nm has (a) frequency $f = c/\lambda = 7.71\times10^{14}$ Hz, and (b) energy $E = hf = 5.11\times10^{-19}$ J $= 3.19$ eV.

(c) For a recession velocity of $v = 3000$ km/s, the line will be redshifted by

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = 0.00667,$$

so the wavelength will increase by a factor of 1.00667, to 392 nm.

3. (a) From the upper graph, the apparent magnitudes m corresponding to colors $B - V = 0.25, 0.5, 1.0$, and 2.5 are approximately 5.6, 7.4, 9.9, and 13.1.

(b) From the lower graph, the absolute magnitudes M corresponding to the same colors are approximately 1.8, 3.6, 6.6, and 10.3.

(c) Using the distance relation

$$m - M \;=\; 5 \log_{10} D\,(\mathrm{pc}) - 5,$$

the four colors yield distance estimates of 59, 57, 46, and 36 pc. The average is 49 pc. Note that there seems to be a trend in distance with color, which comes from the fact (see the slides for lecture 3.1) that the theoretical and observed CMDs don't align perfectly. Nevertheless, the average compares quite well with the accepted value of 47 pc. We'll use $D = 49$ pc in the rest of the solution.

(d) The brightest star still on the main sequence in the Hyades has an apparent magnitude of approximately 4.6, although, as discussed in class, there is considerable uncertainty in selecting it because the brightest blue stars are probably blue stragglers, not true main sequence stars. Given that the absolute magnitude of the Sun is 4.8, the Sun's apparent magnitude at 49 pc would be $4.8 + 5 \log_{10}(49/10) = 8.3$. Thus the brightest star is 3.7 magnitudes brighter than the Sun, meaning that its luminosity is $30\, L_\odot$.

(e) The mass–luminosity relation implies that the mass of that star is $2.3\, M_\odot$.

(f) Hence the age of the cluster is 10 Gyr $(M/M_\odot)^{-3} = 780$ Myr. That's somewhat higher than the accepted value of 600 Myr, but you can see the problem of estimating the turn-off luminosity. In practice, we fit an entire theoretical main sequence to the data, and get the turn-off mass from the theoretical track.

4. (a) In convenient units, $k = 8.62 \times 10^{-5}$ eV/K, so for $T = 10,\ 6000$, and 1.5×10^7 K, respectively, we have $E = kT = 8.6 \times 10^{-4},\ 0.52$, and 1290 eV.

(b) The rms speed of a nucleus is $v_{rms} = \sqrt{3kT/m_H} = 0.50,\ 12$, and 609 km/s for the three temperatures of interest here.

(c) The ratio of atoms in the fourth to the first (ground) state is

$$\begin{aligned} r_{41} &= \frac{g_4}{g_1}\, e^{-(E_4 - E_1)/kT} \\ &= 16\, e^{-12.75\,\mathrm{eV}/kT}. \end{aligned}$$

For the above three temperatures, $r_{41} = 0,\ 3.1 \times 10^{-10}$, and 15.8.

(d) The ionization fraction is determined by the Saha equation:

$$\frac{X^2}{1 - X} \;=\; \frac{1}{n\Lambda^3}\, e^{-\chi/kT} \;\equiv\; A,$$

where $\chi = 13.6$ eV and

$$\Lambda = 7.5 \times 10^{-10}\ \mathrm{m} \left(\frac{T}{10^4\ \mathrm{K}}\right)^{-1/2}.$$

For the three temperatures, $A = 0,\ 8.5 \times 10^{-12}$, and 2.8×10^5, so $X = \frac{1}{2}(-A + \sqrt{A^2 + 4A}) = 0,\ 2.9 \times 10^{-6}$, and 1.0.