PHYS 231: Introductory Astrophysics

Winter 2020

Homework #1 solutions

1. (a) The parallax is

$$p \text{ (arcsec)} = \frac{1}{D \text{ (pc)}},$$

so for $D = 1.8$ pc, $p = 0.56$ arcsec. The proper motion is

$$\mu \text{ (arcsec/yr)} = \frac{v_t \text{ (km/s)}}{4.74\, D \text{ (pc)}},$$

so if $v_t = 90$ km/s, $\mu = 10.5$ arcsec/yr.

(b) The angular size (in radians) of an object of diameter d at distance D is

$$\theta = \frac{d}{D},$$

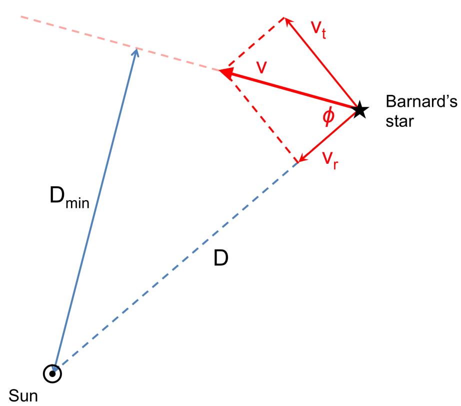
so if $d = 2.7 \times 10^5$ km, $\theta = 4.9 \times 10^{-9}$ radians $= 1.0 \times 10^{-3}$ arcsec.

(c) If $v_r = 110$ km/s and v_t is as in part (a), then the total speed is

$$v = \sqrt{v_t^2 + v_r^2} = 142 \text{ km/s}$$

and the direction angle ϕ (see figure at right) is

$$\phi \;=\; \tan^{-1} \frac{v_t}{v_r} \;=\; 0.69 \text{ rad} \;=\; 39^\circ.$$



(d) From the figure, the minimum distance between Barnard's star and the Sun will be

$$D_{min} \;=\; D \sin\phi \;=\; 1.1 \text{ pc},$$

and this will occur in

$$t \;=\; \frac{D \cos\phi}{v} \;=\; 9600 \text{ years}.$$

2. (a) The comet has zero energy as it falls inward, so its speed v at distance r is v_C, where

$$v_C^2 = \frac{2GM_\odot}{r}$$

(so v is just the escape speed at this radius. For $r = 1$ AU, we find $v_C = 42.1$ km/s.

(b) Earth's orbital speed in a circular orbit at 1 AU is $v_\oplus$:

$$v_\oplus = \sqrt{\frac{GM_\odot}{r}} = 29.8 \text{ km/s}.$$

The relative speed of the comet relative to Earth is

$$v_{rel} = \sqrt{v_C^2 + v_\oplus^2} = 51.6 \text{ km/s}.$$

(c) The mass of the comet, of radius R and density ρ, is

$$M_C = \tfrac{4\pi}{3}\rho R^3 = 2.62 \times 10^8 \text{ kg},$$

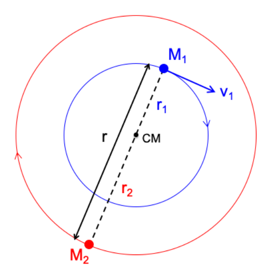
so the kinetic energy in the relative motion is

$$\begin{aligned} \tfrac{1}{2} m_C v_{rel}^2 &= 3.5 \times 10^{17} \text{ J} \\ &= 87 \text{ megatons,} \end{aligned}$$

which is a lot of energy!

3. For the orbital geometry shown in the figure at right, we have

$$\begin{aligned} M_1\, r_1 &= M_2\, r_2 \\ r &= r_1 + r_2 = r_1\left(1 + \frac{M_1}{M_2}\right) \\ P^2 &= \frac{4\pi^2 r^3}{G(M_1+M_2)} \quad \text{(Kepler III)} \\ \frac{v_1^2}{r_1} &= \frac{GM_2}{r^2} \quad \text{(circular motion)} \end{aligned}$$



The third equation implies

$$r^3 = \frac{G(M_1+M_2)P^2}{4\pi^2},$$

while the fourth gives

$$r = \frac{GM_2^2}{v_1^2(M_1+M_2)}.$$

Combining these equations to eliminate r, we find

$$\frac{M_2^3}{(M_1+M_2)^2} = \frac{Pv_1^3}{2\pi G},$$

which is equation 2.44 in Maoz with $\sin i = 1$.

In our case we know $M_1 = 20\, M_\odot$, $P = 6$ days, and $v_1 = 121$ km/s, so setting $M_2 = \mu M_1$, we find

$$\frac{\mu^3}{(1+\mu)^2} = \frac{Pv_1^3}{2\pi G M_1} = 0.0551.$$

The solution to this nonlinear equation (obtained by trial and error, or by writing a short program) is $\mu = 0.50$, so

$$M_2 = 10\, M_\odot.$$

Plugging this into Kepler's third law, we find

$$r = 0.2 \text{ AU}.$$

4. (a) The cluster has

$$m_V - M_V = 5\log_{10} D \text{ (pc)} - 5,$$

so, for $m_V = 15.8$ and $M_V = 0.7$, we find

$$D \;=\; 10^{20.1/5} \text{ pc} \;=\; 10.5 \text{ kpc}.$$

(b) At distance D, proper motion of $\mu = 7.7 \times 10^{-3}$ arcsec/yr translates to transverse velocity

$$v_t \;=\; 4.74\, D \text{ (pc} \mu \;=\; 382 \text{ km/s}.$$

The radial velocity v_r is given by

$$\frac{v_r}{c} \;=\; \frac{\Delta\lambda}{\lambda} \;=\; 3.6 \times 10^{-4},$$

so $v_r = 108$ km/s. The total velocity is therefore

$$v \;=\; \sqrt{v_t^2 + v_r^2} \;=\; 397 \text{ km/s}.$$

(c) The mass required to keep M15 in a circular orbit at this speed at distance $R = 9.3$ kpc from the Galactic center is

$$M \;=\; \frac{v^2 R}{G} \;=\; 6.8 \times 10^{41} \text{ kg} \;=\; 3.4 \times 10^{11}\, M_\odot.$$

5. (a) By Wien's law, the spectrum f_λ of a blackbody of temperature T peaks at wavelength

$$\begin{aligned} \lambda_{max} &= \frac{2.9 \times 10^{-3} \text{ m}}{T \text{ (K)}} \\ &= 2.99 \times 10^{-7} \text{ m} \\ &= 299 \text{ nm}. \end{aligned}$$

(b) By Stefan's law, the flux at the surface is

$$f \;=\; \sigma T^4 \;=\; 5.02 \times 10^8 \text{ W/m}^2.$$

(c) The luminosity is

$$L = 4\pi R^2 f \;=\; 2.2 \times 10^{28} \text{ W} \;=\; 58\, L_\odot.$$

(d) The parallax is 0.13 arcsec, so the distance is $D = 7.7$ pc and

$$m_V - M_V = 5\log_{10} D \text{ (pc)} - 5,$$

so for $m_V = 0.03$ we have $M_V = 0.6$.

(e) For a telescope of limiting magnitude 20, the maximum distance at which Vega can still be seen is D_{max}, where

$$20 - M_V = 5\log_{10} D_{max} - 5,$$

so

$$D_{max} \;=\; 10^{24.6/5} \text{ pc} \;=\; 76 \text{ kpc}.$$