PHYS 305: Computational Physics II

Winter 2022

Homework #4

(Due: February 21, 2022)

Each problem is worth 10 points. Upload your solutions to Learn with a title including PHYS 305 and the Homework number. The PDF upload should contain all discussion, results, and graphs requested, and files containing Python scripts for all programs written.

1. Incorporate variable time steps, as implemented in the solution to Exercise 6.3, into the following N-body calculation. Create a system of $N = 25$ identical particles with total mass 1, initially distributed in a homogeneous sphere of radius 1, with initial speeds $v = 0.1$ distributed isotropically in direction (as discussed in class). Choose an initial `numpy` random seed of 12345, and be sure to place the system in the center of mass frame. Use a softening parameter $\epsilon = 0.01$, and run the calculation until $t = 20$ time units.

   (a) Implement variable steps by multiplying the parameter τ returned by the integrator by a time step parameter η (set by default and/or on the command line), which can be varied to control the accuracy of the calculation. Print out the time and energy error $|E - E_0|$ (where E_0 is the initial total energy) at intervals of 0.5 time unit, and find a value of η that ensures an energy error of less than 10^{-4} throughout the run. How many total time steps does the calculation take?

   (b) Also at intervals of 0.5 time unit, check for bound pairs of particles with relative energy less than E_0/N. If you find any, print the particles involved and their relative orbital energy and period.

   (c) Plot as functions of time (on the same graph) (i) the total energy, (ii) the total kinetic energy, (iii) the total potential energy U, and (iv) the "virial radius" R_v defined by

   $$R_v = -\frac{GM^2}{2U},$$

   where M is the total system mass. Also plot, on a linear (x) – log (y) plot, the time variation of the time scale τ.

   (d) Repeat part (a) using a fixed timestep δt. Can you find a value of δt that ensures an energy error of less than 10^{-4} throughout the run? How many total time steps does the calculation take?

2. (a) Do Exercise 6.5 on the course web page. Specifically, implement a step-doubling scheme for the two-body problem with the given initial conditions, and find the value of the control parameter dE_{tol} needed to ensure an energy error of less than 10^{-4} throughout the entire run.

3. Repeat problem 1(a) using a step doubling scheme. Find the value of dE_{tol} needed to ensure an energy error of less than 10^{-4} throughout the entire run, and compare the total number of steps to the values found in parts 1(a) and 1(d).