PHYS 511: Electromagnetism

Homework #1

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January 17, 2019

1. **(Jackson 1.1)** Use Gauss's theorem to prove the following:

   (a) Any excess charge placed on a conductor moust lie entirely on its surface. (A conductor by definition contains charges capable of moving freely under the action of applied electric fields.)
   Solution:
   Lets assume that the charge lies inside the volume of the conductor. Making a gaussian surface that lies within a volume of conductor and encloses this assumed charge would imply there is finite flux through this sufrace and hence electric field. But electric field inside a conductor is not possible because otherwise the charges would move and we would no longer have static equilibrium. Thus by contradiction, there can be no charge inside the volume of conductor. □

   (b) A closed, hollow conductor shields its interior from fields due to charges outside, but does not shield its exterior from the fields due to thcarges placed inside it.
   Solution:
   Lets consider two cases, when there is charge inside the conductor and when there is charge outside the conductor. In the first case if we take a gaussian surface that completely encloses the hollow conductor, by gauss's law we get finite electric field at any arbitrary point outside the hollow conductor. Thus the conductor doesn't shield the outside from electric field.
   In the second case, when charge is outside. The flux through the gaussian surface enclosing the conductor is zero as there is no charge inside. Since the electric field outside only induces the charge on the surface of the conductor. There can't be field inisde the hollow conductor. □

   (c) The electric field at the surface of a conductor is normal to the surface and has a magnitude $\frac{\sigma}{\epsilon_0}$ where σ is the charge density per unit area on the surface.
   Solution:
   Let us assume an arbitrary gaussian surface parallel and very close to the surface of conductor. In such a case the field at every point on the surface is equal and normal to the the plane of this surface. Using gaussian law for this

$$\begin{aligned}\int_A \boldsymbol{E}\cdot d\boldsymbol{A} &= \frac{q}{\epsilon_0}\\ EA &= \frac{\sigma A}{\epsilon_0}\\ \implies E &= \frac{\sigma}{\epsilon_0}\end{aligned}$$

   This shous that the electric field near the surface of conductor is normal to the surface and has magnitude of σ/ϵ_0.
   □

2. **(Jackson 1.3)** Using the Dirac delta functions in the appropriate corrdinates, express the following charge distributions as three-dimesional charge densities $\rho(\boldsymbol{x})$

   (a) In spherical coordinates, a charge Q uniformaly distributed over a spherical shell of radious R.
   Solution:

Since the total charge Q is uniformly distributed over the surface of shell, and the total surface area of shell is $4\pi R^2$ we have, total surface density given by

$$\frac{Q}{4\pi R^2}$$

Now in the entire space, the only place this surface charge can be found is at the surface of sphere of radious R thus the total charge density over all space becomes

$$\rho(\boldsymbol{x}) = \rho(r, \theta, \phi) = \frac{Q}{4\pi R^2}\delta(r - R)$$

This gives the total charge density over all space if the charge is in spherical shell of radius R . $\square$

(b) In cylindrical coordinates, a charge λ per unit lentgh uniformly distributed over a culindrical surface of radious b.
Solution:
Let us consider a arbitrary length of the cylindrical surface l, with radius b. Now, the total surface area of this arbitrary cylindrical section is $V = 2\pi bl$. The total surface density of charge is for some charge Q is

$$\frac{Q}{2\pi bl}$$

The only place this charge can be found in all of space is for locations where $r = b$ (in cylindrical coordinate). Thus the total charge density over all aspace becomes

$$\rho(\boldsymbol{x}) = \rho(r, \phi, z) = \frac{Q}{2\pi bl}\delta(b - r) = \frac{\lambda}{2\pi b}\delta(b - r)$$

This gives the total charge density over all space if the charge is in cylindrical surface of radius bof radius b provided the linear charge density is λ. $\square$

(c) In cylindrical coordinates, a charge Q spread uniformly over a flat circular disc of negligible thickness and radius R.
Solution:
The total area of the circular disc is πR^2. The surface charge density of for this disk is

$$\frac{Q}{\pi R^2}$$

. Now since the disk is negligible thickness the total only place where this charge resides in entire space is where $z = 0$ and $r \leq R$. Thus the total volume charge density over entire space vbecomes

$$\rho(\boldsymbol{x}) = \rho(r, \phi, z) = \frac{Q}{\pi R^2}\delta(z)\mathcal{H}(r - R)$$

where $\mathcal{H}(x) = 1$ if $x < 0, 0$ otherwise.

$\square$

(d) The same as (2c), but using spherical coordinates.
Solution:
From (2c), we can make the change of coordinate as $z = r\cos(\theta)$. Substutin that in denta function, and using the property fo denta function

$$\delta(\alpha x) = \frac{1}{|\alpha|}\delta(x)$$

the total density becomes

$$\rho(\boldsymbol{x}) = \rho(r, \phi, \theta) = \frac{Q}{\pi R^2}\delta(r\cos\theta)\mathcal{H}(r - R) = \frac{Q}{\pi R^2}\frac{1}{r}\delta(\cos\theta)\mathcal{H}(r - R)$$

where $\mathcal{H}(x) = 1$ if $x < 0, 0$ otherwise. $\square$