Physics 201 Finals Additional Review Exercises

1. The negative pion π^- is an unstable particle with an average lifetime of 2.6 x 10^{-8} s (measured in the rest frame of the pion).

a. If the pion is made to travel at very high speed relative to a laboratory, its average lifetime is measured in the laboratory to be 4.2 x 10^{-7} s. Calculate the speed of the pion expressed as a fraction of c.

b. What distance, measured in the laboratory frame, does the pion travel during its average lifetime ?

c. What distance, measured in the pion's frame, does the pion travel during its average lifetime ? Do *not* use length contraction expression here.

d. Using the length contraction expression, determine the distance the laboratory appears to move through, as viewed by the pion during its lifetime. Compare to the answer in (b).

Solution:

(a) IDENTIFY and SET UP: $\Delta t_0 = 2.60\times10^{-8}$ s; $\Delta t = 4.20\times10^{-7}$ s. In the lab frame the pion is created and decays at different points, so this time is not the proper time.

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ says $1-\dfrac{u^2}{c^2} = \left(\dfrac{\Delta t_0}{\Delta t}\right)^2$

$$\frac{u}{c} = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{2.60\times10^{-8}\text{ s}}{4.20\times10^{-7}\text{ s}}\right)^2} = 0.998;\ u = 0.998c$$

EVALUATE: $u < c$, as it must be, but u/c is close to unity and the time dilation effects are large.

(b) IDENTIFY and SET UP: The speed in the laboratory frame is $u = 0.998c$; the time measured in this frame is Δt, so the distance as measured in this frame is $d = u\Delta t$

EXECUTE: $d = (0.998)(2.998\times10^8\text{ m/s})(4.20\times10^{-7}\text{ s}) = 126\text{ m}$

EVALUATE: The distance measured in the pion's frame will be different because the time measured in the pion's frame is different (shorter).

c. In the pion's frame, it would have moved a distance of

$$d = (0.998)(3 \times 10^8\text{ m/s})(2.6 \times 10^{-8}\text{ s}) = 7.8\text{ m}$$

d. The distance as measured in the pion's frame is given by the length contraction expression:

$$l = l_0\sqrt{1-u^2/c^2}$$

where l_o is the proper length = 126 m. Thus l = 126m $(1-0.998^2)^{1/2}$ which is very close to the value in (c).

2. The photoelectric work function of potassium is 2.3 eV. If light having a wavelength 250 nm falls on potassium, determine:

a. the stopping potential, units of volts
b. KE_{max} , in units of eV, of the most energetic electrons ejected
c. speed of those electrons
d. What is the speed of these electrons when the intensity of the incident light is doubled ?

Solution:

IDENTIFY and SET UP: $eV_0 = \frac{1}{2}mv_{max}^2$, where V_0 is the stopping potential. The stopping potential in volts equals eV_0 in electron volts. $\frac{1}{2}mv_{max}^2 = hf - \phi$.

EXECUTE: **(a)** $eV_0 = \frac{1}{2}mv_{max}^2$ so

$$eV_0 = hf - \phi = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{250\times10^{-9}\ \text{m}} - 2.3\ \text{eV} = 4.96\ \text{eV} - 2.3\ \text{eV} = 2.7\ \text{eV}$$

The stopping potential is 2.7 electron volts.

(b) $\frac{1}{2}mv_{max}^2 = 2.7\ \text{eV}$

(c) $v_{max} = \sqrt{\frac{2(2.7\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = 9.7\times10^{5}\ \text{m/s}$

d. Nothing happens, because doubling the intensity means there will just be twice as more electrons ejected.

3. A triply ionized beryllium ion Be^{3+} (a beryllium atom with three electrons removed) behaves very much like a hydrogen atom except that the nuclear charge is four times as great.

a. What is the ground state energy of this ion, in units of eV ?
b. What is its ionization energy, in units of eV ?
c. What is the wavelength of a photon emitted when the ion undergoes a transition from n = 2 to n =1 ? What is its frequency, in units of Hz ?

Solution:

a. **EXECUTE:** $E_n = -\frac{1}{\epsilon_0}\frac{me^4}{8n^2h^2}$ (hydrogen) becomes

$$E_n = -\frac{1}{\epsilon_0}\frac{m(Ze^2)^2}{8n^2h^2} = Z^2\left(-\frac{1}{\epsilon_0}\frac{me^4}{8n^2h^2}\right) = Z^2\left(-\frac{13.60\text{ eV}}{n^2}\right) \text{ (for Be}^{3+}\text{)}$$

The ground-level energy of Be^{3+} is $E_1 = 16\left(-\frac{13.60\text{ eV}}{1^2}\right) = -218\text{ eV}$.

b. The ionization energy is the difference between the atom's energy when n = 1 (ground state) and the energy when n = ∞ (free particle). So the ionization energy must be **218 eV**.

c.

(c) SET UP: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ just as for hydrogen but now R has a different value.

EXECUTE: $R_H = \frac{me^4}{8\epsilon_0 h^3 c} = 1.097\times10^7\text{ m}^{-1}$ for hydrogen becomes

$$R_{Be} = Z^2\frac{me^4}{8\epsilon_0 h^3 c} = 16(1.097\times10^7\text{ m}^{-1}) = 1.755\times10^8\text{ m}^{-1} \text{ for Be}^{3+}.$$

For $n = 2$ to $n = 1$, $\frac{1}{\lambda} = R_{Be}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 3R/4$.

$$\lambda = 4/(3R) = 4/(3(1.755\times10^8\text{ m}^{-1})) = 7.60\times10^{-9}\text{ m} = 7.60\text{ nm}.$$

You can also solve this using $\Delta E = hc/\lambda$

To find frequency, use E = hf where h = Planck's constant. You'd need to convert E to Joules and solve for f in units of Hz.

4. Suppose an electron is trapped in a one-dimensional box of length L and the electron's energy equals the absolute value of the ground state of the electron in a hydrogen atom.

a. Determine the length of the box.
b. What would be its first excited energy, in units of eV ?
c. If an electron excited to its first excited energy falls to the ground state, what is the wavelength of the emitted photon, in units of nm ?
d. What is the probability that, in the ground state, that the electron is located between x=0 and x=L/4 ?

Solution:

a. The electron can be modeled as a free particle in an infinite square well so its energy spectrum is given by

$$E_n = n^2h^2/(8mL^2)$$

where n =1 (ground state) and n = 2 (1st excited state). Using the absolute value of hydrogen's ground state energy (13.67 eV) as E_1, one can solve for L:

$$L = \frac{h}{\sqrt{8mE_1}} = \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})}{\sqrt{8(9.11\times10^{-31}\ \mathrm{kg})(13.6\ \mathrm{eV})(1.602\times10^{-19}\ \mathrm{J/eV})}} = 1.66\times10^{-10}\ \mathrm{m}.$$

b. Since $E_n = E_1n^2$, then $E_2 = (13.67\mathrm{eV})2^2 = 54.68$ eV

c. $\Delta E = hc/\lambda = E_2 - E_1 = (54.68 - 13.67)$ eV. You need to convert eV to Joules and solve for λ in nm. Answer: $\lambda = 31$ nm.

d. In the ground state, the normalized wavefunction is $\Psi_1 = [\sqrt{(2/L)}\,]\sin(\pi x/L)$. To find the probability that the electron is between x = 0 and x = L, one integrates $|\,\Psi_1\,|^2$ over that region:

$$\frac{2}{L}\int_0^{L/4}\sin^2\frac{\pi x}{L}dx = \frac{2}{L}\int_0^{L/4}\frac{1}{2}\left(1-\cos\frac{2\pi x}{L}\right)dx = \frac{1}{L}\left(x-\frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)_0^{L/4} = \frac{1}{4}-\frac{1}{2\pi},\ \text{which is about 0.0908.}$$