

Solution to Midterm 1

1 a) $A = 10 \text{ cm}$ (from initial condition)

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 10 \text{ rad/s}$$

$$x(t=0) = A = A \cos \phi \Rightarrow \cos \phi = 1 \Rightarrow \phi = 0$$

$$v(t=0) = -A\omega \sin \phi = 0 \Rightarrow \sin \phi = 0 \Rightarrow \phi = 0$$

$$\Rightarrow x(t) = 10 \text{ cm} \cos \omega t$$

b) $v(t) = \frac{dx}{dt} = -\omega (10 \text{ cm}) \sin(\omega t)$

Earliest $v_{\max} = -\omega (10 \text{ cm}) \Rightarrow \sin \omega t = 1$

$$\Rightarrow \omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega} = \frac{\pi}{2(10/\text{s})} = 0.157 \text{ s}$$

$$a(t) = -\omega^2 x = -\omega^2 A \cos \frac{\pi}{2} = 0$$

c) $E = \frac{1}{2} k A^2 = \frac{1}{2} (200) (0.1)^2 = 1 \text{ J}$

d) $U = \frac{1}{2} k x^2 = \frac{1}{2} (200) (0.05)^2 = 0.25 \text{ J}$

$$K = E - U = 1 \text{ J} - 0.25 \text{ J} = 0.75 \text{ J}$$

2. $y(x,t) = 15 \text{ cm} \cos(3x - 30\pi t)$

a) $A = 15 \text{ cm}$, $k = 3/\text{cm}$, $\omega = 30\pi/\text{s}$

$$f = \frac{\omega}{2\pi} = \frac{30\pi}{2\pi} = 15 \text{ Hz} \quad T = \frac{1}{f} = \frac{1}{15} \text{ s} = 0.067 \text{ s}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{3} \text{ cm} = 2.09 \text{ cm}$$

b) $v = \frac{\omega}{k} = \frac{30\pi/\text{s}}{3/\text{cm}} = 10\pi \text{ cm/s} = 31.42 \text{ cm/s}$

c) $v = \sqrt{\frac{F}{\mu}} \Rightarrow F = v^2 \mu$, $v = 0.3142 \text{ m/s}$
 $\mu = \frac{0.25 \times 10^{-3} \text{ kg}}{1 \times 10^{-2} \text{ m}} = 0.025 \frac{\text{kg}}{\text{m}}$
 $F = (0.3142)^2 (0.025)^2 = 2.47 \times 10^{-3} \text{ N}$

d) $P = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} (0.025) (30\pi)^2 (0.15)^2 (0.3142) = 0.78 \text{ W}$

e) $x_{\text{antinode}} = \frac{\lambda}{4} = \frac{2.09 \text{ cm}}{4} = 0.523 \text{ cm}$
 $x_{\text{node}} = \lambda/2 = 1.045 \text{ cm}$